EAST-WEST

Journal of **Economics and Business**
Vol. XIII – 2010, No 1

How useful is the LAVE method?

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Abstract

We show that the findings of Mercurio and Spokoiny (2004) concerning their alternative LAVE approach for volatility estimation are not necessarily true for another type of volatile time series, as far as comparison to the usual GARCH(1,1) process is concerned. However, we propose another use of LAVE for the purpose of level, not volatility, modeling – and in our case it turns out to be successful.

Keywords: LAVE, GARCH, ARIMA, volatility, emerging markets, stock exchange index

JEL Classification: C22, C53

1. INTRODUCTION AND MOTIVATION

For managing risk that we are faced with at the stock market, volatility modeling has become the leading issue. Although the main part of attention in the field of volatility research is focused on the recognized GARCH models and their modifications (see for example survey of Li, Ling and McAleer, 2002), a different approach to volatility modeling was proposed by Mercurio and Spokoiny (2004). They presented the so-called LAVE (*Locally Adaptive Volatility Estimate*) method, and by applying it to nine USD exchange-rate time series they showed that it outperformed the usual GARCH(1,1) model.

This idea is quite attractive – especially because of its simplicity, regular simultaneous checking with self-adapting, and the fact that no assumptions about parametric structure are needed. Thus, the size of a sample is not so crucial any more, and since the LAVE method aims at modeling the local dynamics of the volatility process, it is particularly appropriate for the short-term forecasting.

Therefore, we wanted to find out, whether this alternative method is valid also for some other type of volatile data. We applied the LAVE approach to daily return time series of leading stock market indexes from five Central European emerging markets (i.e. Czech PX, Hungarian BUX, Polish WIG20, Slovakian SAX and Slovenian SBI20) as well as from two developed European financial markets (i.e. British FTSE100 and German DAX). It turned out that in general the dominance of LAVE approach over GARCH(1,1) model cannot be straightforwardly assumed for every type of time series.

However, we made a step further on by extending the applicability of LAVE method, and showed that we can take advantage of LAVE calculations to improve the original time series (i.e. level) models as well. We found that in our case the new ARIMA-LAVE model evidently outperforms the ARIMA-GARCH-M model, both from fitting as well as from forecasting point of view.

We begin by shortly describing the LAVE approach presented by Mercurio and Spokoiny (2004). Next, we propose the use of the LAVE term as an improvement in return (i.e. level) modeling. Then we do the empirical application, consisting of a closer look at the analyzed data, comparison of the LAVE approach to GARCH(1,1) model on the basis of quality of volatility estimation, and comparison of the ARIMA model, refined with the GARCH supplement, to the ARIMA model, where the additional GARCH refinements are substituted by LAVE term, from the quality of level forecasting point of view. Finally, the conclusions are drawn.

2. VOLATILITY MODELING

Volatility, or conditional standard deviation, is an interesting and important feature of a financial time series, since it is a key element of risk. And risk – considered as the probability that our expectations don't come true – has a very high price. The higher the variability of a process the higher this probability. Thus, estimating volatility is essential especially at stock market, where volatility defines risk factor β in a CAPM model, serves as a basis for option pricing and computing VaR (*Value at Risk*), helps defining the confidence intervals of the underlying process more precisely, and the like.

It needs to be stressed that taking volatility into account leads us also to more efficient statistical estimators. Since Mandelbrot's finding (1963) about empirical distributions being more leptokurtic and fat-tailed than normal, scientists have been searching for the most appropriate model. Engle (1982) and Bollerslev (1986) were successful with their (G)ARCH models, because these models imitate the empirical leptokurtic and volatility-clustering processes quite satisfactorily. Since then we have witnessed an outburst of modifications and derivations based on (G)ARCH model. One of the surveys of the (G)ARCH variations is presented for example by Li, Ling and McAleer (2002).

Later, an alternative LAVE method was proposed, and it was shown that it outperformed the most common GARCH(1,1) model. The reader is referred to the article of Mercurio and Spokoiny (2004) to get detailed information on this method, whereas in the following subsection the basic idea of LAVE method is described and commented.

2.1. Locally Adaptive Volatility Estimate (LAVE)

LAVE (*Locally Adaptive Volatility Estimate*) does not assume any parametric form of the volatility process. The main presumption is that the volatility can be approximated by a constant over some interval. Therefore, the basic problem here is finding this interval of homogeneity. With the LAVE approach the model is regularly checked and adapted to the data, meaning that for every time point τ we estimate the past interval of time homogeneity $[\tau - m, \tau]$, over which the volatility σ_τ is nearly constant. Once this interval is defined, we estimate the corresponding volatility, which can then be used for one-step ahead forecasting. It is assumed that the interval of time homogeneity will be extended into the next day implying that volatility remains approximately constant – we therefore consider the volatility to have the characteristics of a martingale.

The underlying process ($R_t = \log(S_t / S_{t-1})$) is modeled as conditional heteroscedastic by

$$R_t = \sigma_t \xi_t, \tag{1}$$

where ξ_t with $t \geq 1$ is a sequence of independent standard Gaussian random variables, and σ_t is volatility, which is in general a predictable random process dependent on the first $t-1$ observations. From multiplicative model we construct an additive one via transformation. Since log-transformation results in highly skewed distribution of the errors ξ_t, we take advantage of power transformation (for every $\gamma > 0$):

$$E(|R_t|^\gamma |\Im_{t-1}) = \sigma_t{}^\gamma E(|\xi|^\gamma |\Im_{t-1}) = C_\gamma \sigma_t{}^\gamma,$$
$$E(|R_t|^\gamma - C_\gamma \sigma_t{}^\gamma |\Im_{t-1})^2 = \sigma_t{}^{2\gamma} E(|\xi|^\gamma - C_\gamma)^2 = \sigma_t{}^{2\gamma} D_\gamma{}^2,$$

where $C_\gamma = E|\xi|^\gamma$ and $D_\gamma{}^2 = Var|\xi|^\gamma$. The process $|R_t|^\gamma$ can be thus represented by

$$|R_t|^\gamma = C_\gamma \sigma_t{}^\gamma + D_\gamma \sigma_t{}^\gamma \varsigma_t \qquad (2)$$

with $\varsigma_t = (|\xi|^\gamma - C_\gamma) / D_\gamma$.

The local time homogeneity approach assumes that the volatility function σ_t is nearly constant within an interval $I = [\tau - m, \tau]$. The process R_t follows the equation (2) with the constant trend that can be estimated by averaging over the interval I as

$$\widetilde{\theta}_I = \frac{1}{|I|} \sum_{t \in I} |R_t|^\gamma,$$

with the conditional standard deviation estimate $\widetilde{v}_I = s_\gamma \widetilde{\theta}_I |I|^{-1/2}$, where $s_\gamma = D_\gamma / C_\gamma$.

Given the observations $R_1, \ldots, R_\tau$ and using the above relations, we try to estimate the parameter value θ_τ by its estimator $\widetilde{\theta}_I$, with properly chosen time interval I, in order to minimize the estimation error.

The logic behind this procedure is as follows. Let I be a time homogenous interval candidate. This means that time homogeneity is expected in I as well as in every subinterval J, $J \subset I$, with estimates of θ_τ over I and J almost coinciding. The aim of this method is practically searching for the largest possible interval I, such that the hypothesis about θ_τ being constant over I is not rejected. For testing this hypothesis, we examine subintervals of the form $J = [\tau - m', \tau],\ m' < m,$ by comparing two different estimates of θ_τ – one is calculated on the interval J and the other one on its complement $K = I \setminus J = [\tau - m, \tau - m']$. With high probability it holds that $|\tilde{\theta}_K - \tilde{\theta}_J| \leq \lambda \sqrt{\tilde{v}_K^{\,2} + \tilde{v}_J^{\,2}}$, λ being sufficiently large. We continue with lengthening the interval I until the hypothesis is rejected.

When conducting the empirical analysis we consider a simple proposal by Mercurio and Spokoiny (2004) about using a regular time grid with step $m_0 \in \mathbb{N}$, that is $t_k = m_0 k,\ k = 1,2,\ldots$, to define the intervals. For a given time point τ, the set $\mathcal{I}$ of interval candidates is defined as

$$\mathcal{I} = \{ I_k = [t_k, \tau] :\ t_k \leq \tau - m_0, \quad k = 1,2,\ldots \}.$$

Next, for every interval I_k we define the set $\mathcal{J}(I_k)$ of testing subintervals $J_{k'} \subset I_k$, such that $J_{k'} = [t_{k'}, \tau]$ for all $t_{k'} > t_k$ belonging to the time grid. The homogeneity of I_k is then tested by comparing the pairs of estimates of $\tilde{\theta}$ for complementary subintervals for all $J \in \mathcal{J}(I_k)$.

Since the grid step m_0 completely determines the intervals, it should be chosen as small as possible to minimize the delay before the LAVE algorithm can detect a change point. However, it should be large enough to ensure stability of the estimates $\tilde{v}$. The value of $m_0 = 10$ was pointed out as a good compromise by Mercurio and Spokoiny (2004). The authors also suggest the use of the smoothing parameter $\lambda = 2.74$ (calculated on the simulated data) and power transformation $\gamma = 0.5$, where they argue that the latter is preferable to the more natural $\gamma = 2.0$, despite the loss of efficiency, since it assures greater normality of the errors.

The described algorithm is simple to program and self-adapting estimation is highly appreciated. It is especially welcome in estimating time series that are subject to possible structural breaks, since they are self-detected and taken care of. Moreover, due to local adapting, long data series are not required any more.

However, when conducting the empirical application on a different type of data, it turned out that the LAVE method is not necessarily more successful in explaining the variation of conditional standard deviation than the usual GARCH(1,1) model (cf. Section 4.2).

Nevertheless, because of its simplicity and no assumptions, we checked whether the LAVE method can be applied in some other way as well. Our proposal, that proved successful in empirical application (cf. section 4.3), is to include the LAVE term into the model for return, i.e. level, modeling. This idea is described in the following section.

3. LEVEL MODELING

Despite increasing importance of volatility modeling in financial area, level modeling shouldn't be forgotten either. It still represents the basis for most of the trading conducted on security markets.

Therefore, we were interested in the Box-Jenkins ARIMA models, refined with volatility terms. First, we propose an ARIMA model with LAVE correction, which is then compared to the Bollerslev's ARIMA model with additional GARCH terms.

3.1. ARIMA-LAVE model

To account for volatility in an ARIMA model, we simply include the conditional standard deviation, as estimated by the LAVE method, as an additional explanatory variable. This is not a true multivariate model, since it is set up only with the underlying variable and its derivation:

$$R_t = \alpha_0 + \alpha_1 R_{t-1} + \alpha_2 \varepsilon_{t-1} + \alpha_3 LAVE_t + \varepsilon_t \,, \tag{3}$$

where $LAVE = \sqrt{\widetilde{\theta}_I} = \sqrt{\frac{1}{|I|}\sum_{i \in I} |R_i|^2}$ with I being the corresponding interval of homogeneity.

4. EMPIRICAL APPLICATION

In the empirical part of their paper Mercurio and Spokoiny (2004) analyzed daily exchange rates of the U. S. dollar (USD) against 9 currencies from financially developed countries: Australian dollar (AUD), British pound (GPB), Canadian dollar (CAD), Danish krone (DKK), Japanese yen (JPY), Norwegian krone (NOK), New Zealand dollar (NZD), Swiss franc (CHF) and Swedish krona (SEK).

Besides exchange rates, interest rates, and inflation, is also stock index return one of the most typical financial time series. Therefore, our analysis was conducted on the data for stock index daily returns from five Central European emerging stock markets and two developed stock markets for the period from the beginning of the year 1996 till the end of the year 2005.

4.1. The data

The stock indexes included in the analysis were the following:

- PX is the leading index of Prague Stock Exchange (Burza cenných papírů Praha). It is calculated as a market-capitalization-weighted price index of (currently) 9 stocks with base date April 5th, 1994.
- BUX is the leading index of Budapest Stock Exchange (Budapesti értéktőzsde). It is calculated as a market-capitalization-weighted performance index of 12 stocks with base date January 2nd, 1991.
- WIG20 is the leading index of Warsaw Stock Exchange (Giełda papierów wartościowych w Warszawie). It is calculated as a market-capitalization-weighted price index of 20 stocks with base date April 16th, 1994.
- SAX is the leading index of Bratislava Stock Exchange (Burza cenných papierov v Bratislave). It is calculated as a market-capitalization-weighted total return index of (currently) 5 stocks with base date September 14th, 1993.
- SBI20 is the leading index of Ljubljana Stock Exchange (Ljubljanska borza). It is calculated as a market-capitalization-weighted price index of (currently) 15 stocks with base date January 1st, 1994.

These data differ from the exchange rates that were analyzed by Mercurio and Spokoiny (2004) from two aspects. One, the stock index returns can be considered more volatile than exchange rates (absence of the authority such as the central bank, more frequent trading, more unexpected events resulting in extreme stock prices,…). Two, the emerging markets themselves are supposed to be more volatile than the developed (Aggarwal, Inclan, Leal (1999)). To control for the last distinction we included also two "developed" leading stock indexes:

- FTSE100 is the leading index of London Stock Exchange. It is calculated as a market-capitalization-weighted price index of 100 stocks with base date December 30th, 1983.
- DAX is the leading index of German Stock Exchange (Deutsche Börse). It is calculated as a market-capitalization-weighted return index of 30 stocks with base date December 30th, 1987.

The descriptive statistics of these seven index daily return time series, calculated as $r_t = \ln(P_t / P_{t-1})$, where P_t is the index value on the day t, are summarized in Table 1. The period analyzed is from the beginning of January 1996 till the end of December 2005 for all seven time series in question.

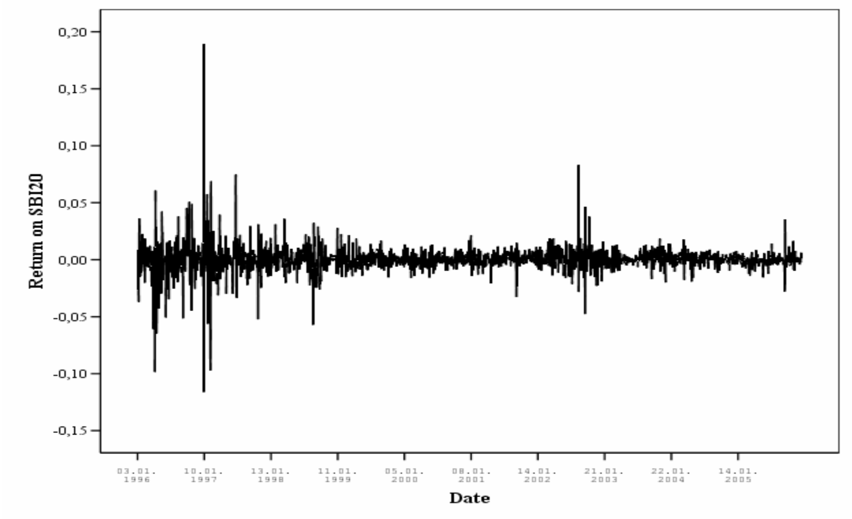
The time series of daily return on SBI20 and its histogram are graphed in Figures 1 and 2, respectively. The other time series exhibit similar patterns of changing volatility of the underlying time series and peakedness of the distribution. Throughout the whole paper the figures are thus prepared for SBI20. If not stressed differently, the presented characteristics are not much different for the other analyzed indexes.

Table 1: Descriptive statistics of daily index return time series

Index	Number of observations	Minimum	Maximum	Mean	Standard deviation	Skewness	Kurtosis
PX	2502	-0,0708	0,0582	0,000486	0,0123	-0,28	5,12
BUX	2486	-0,1803	0,1362	0,001044	0,0184	-0,93	15,57
WIG20	2500	-0,1032	0,0765	0,000480	0,0175	-0,18	5,71
SAX	2426	-0,1148	0,0957	0,000407	0,0140	-0,40	9,11
SBI20	2498	-0,1161	0,1893	0,000481	0,0115	0,89	47,13
FTSE100	2524	-0,0559	0,0590	0,000168	0,0113	-0,14	5,53
DAX	2527	-0,0665	0,0755	0,000345	0,0159	-0,15	5,35

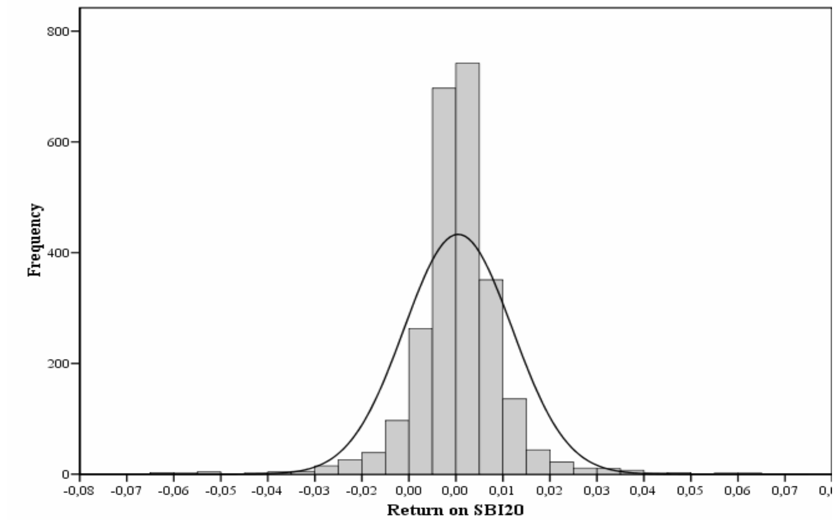
Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

Figure 1: Daily return on Slovenian leading stock exchange index SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

Figure 2: Histogram for daily return on Slovenian leading stock exchange index SBI20 (with normal curve)



Source: Own calculations based on the data from Ljubljana Stock Exchange

From the presented data we can see that these time series are far from being normally distributed. If the distributions of the returns were normal, the kurtosis would be equal to 3, whereas we get the kurtosis coefficient values from 5,12 for PX up to 47,13 for SBI20, which means that the probability distributions are highly leptokurtic, as is also seen from Figure 2.

If we pay some more attention to Figure 1, where the line chart of daily return on SBI 20 time series is shown, we notice clearly visible volatility clusters indicating the alternating periods of high and low volatility.

As expected, we see that the average daily return on FTSE100 (and DAX) is considerably lower than mean daily returns on emerging market stock indexes. This was anticipated since emerging markets are supposed to be less efficient (for example Claesens, Dasgupta, Glen, 1995, Deželan, 2000, Mramor, 2003, Mramor, Umberger, Hieng, 2006) and therefore more prone to possibilities of earning some extra profit. Also the standard deviation of FTSE100 daily returns is lower than the others, but surprisingly, this is not true for DAX.

Using standard deviation as a measure of variability, we can observe changing of yearly variability for each index, as presented in Table 2.

Table 2: Yearly standard deviation of daily return time series for the seven indexes

Year	PX	BUX	WIG20	SAX	SBI20	FTSE100	DAX
1996	0,006879	0,015078	0,015794	0,011917	0,018856	0,005931	0,008060
1997	0,010184	0,025235	0,018191	0,012833	0,023158	0,009524	0,014750
1998	0,015049	0,030640	0,026401	0,015218	0,011015	0,013386	0,018162
1999	0,013205	0,019975	0,018105	0,019547	0,006732	0,011292	0,013844
2000	0,015698	0,016955	0,021244	0,014615	0,005481	0,012025	0,015095
2001	0,014097	0,014175	0,018434	0,013953	0,005773	0,013261	0,018045
2002	0,013904	0,014668	0,015298	0,014429	0,011103	0,017343	0,025192
2003	0,009632	0,011243	0,014136	0,011468	0,005893	0,012216	0,019768
2004	0,009754	0,010727	0,010865	0,010810	0,004972	0,006514	0,009919
2005	0,011074	0,015062	0,010764	0,012646	0,005227	0,005510	0,007624

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

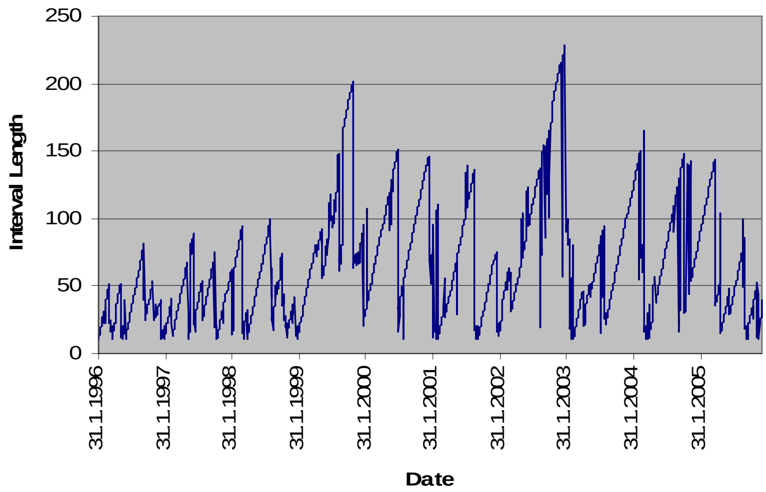
We notice that yearly standard deviation for FTSE100 and DAX is changing in the same sense – from low in 1996 to very high in 2002 and back to low in the last two observed years. Very low return variability in the last years is also observed for SBI20, and moderately low for WIG20 and SAX. When analyzing yearly variability of BUX daily returns, we can conclude that it is lower in the second half of the observed period (after 2001), whereas the yearly variability of daily returns on PX has become high again in the last two observed years. Roughly it looks like the yearly variability of daily index returns is decreasing in emerging markets (excluding PX) indicating that the initial briskness of trading on various arbitrage possibilities is slowly coming to an end.

One of the most obvious characteristic of all analyzed time series we have observed is that they are highly leptokurtic; this is a characteristic of a great majority of financial time series that is supposed to be most nicely captured by GARCH method, as far as volatility modeling is concerned. We will check whether LAVE method serves better than GARCH also in our case – as it was shown by Mercurio and Spokoiny (2004) for their data.

4.2. Volatility modeling

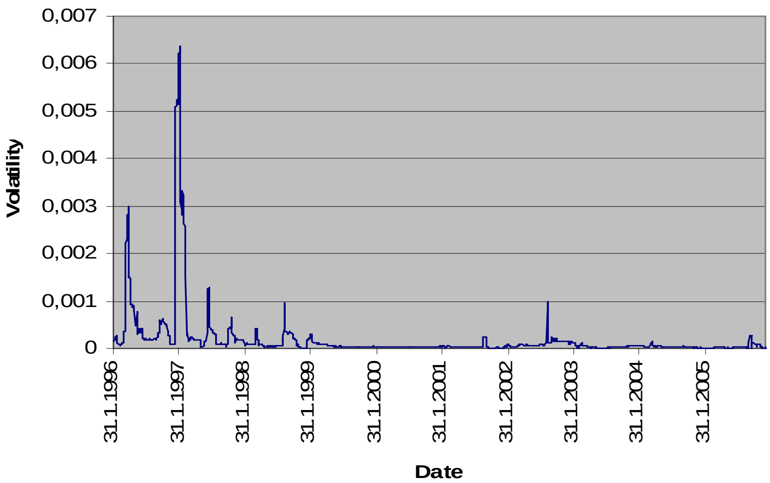
The results of LAVE estimation of intervals of time homogeneity and their volatility for Slovenian stock exchange index SBI20 are presented in Figures 3a and 3b.

Figure 3a: Estimated length of interval of time homogeneity for returns on SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

Figure 3b: Estimated corresponding volatility for returns on SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

It can be observed that during the periods of lower volatility the intervals become larger and the other way around. The estimated volatility nicely coincides with the basic picture of daily return on SBI20 as well.

Comparison of LAVE with GARCH Model

To compare the LAVE performance with the more recognized GARCH approach, we do the volatility estimation also by one of the most common and most widely used models – GARCH(1,1) model by Bollerslev (1986).

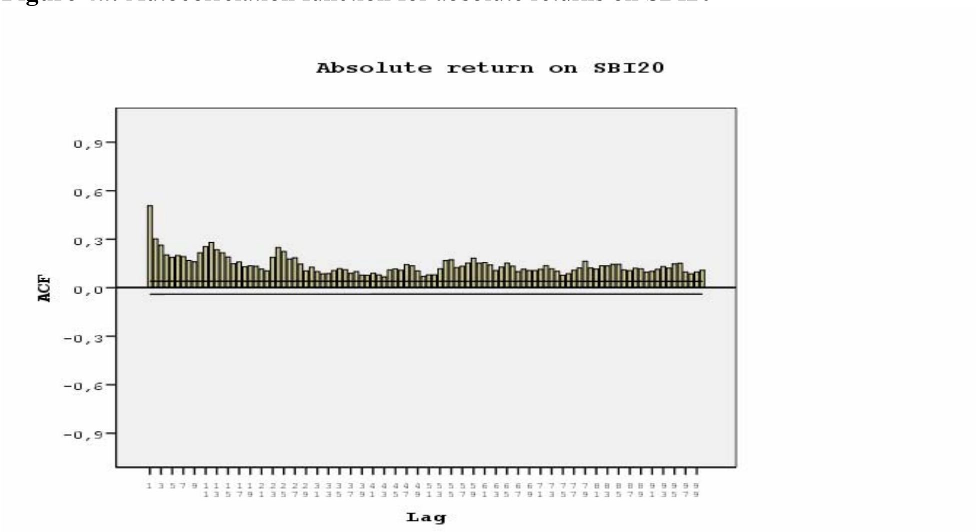
The estimated GARCH(1,1) model for the whole analyzed period for SBI20 index daily return volatility is

$$\sigma_t^2 = \underset{\begin{pmatrix} z = 5{,}97 \\ p = 0{,}0000 \end{pmatrix}}{2{,}8215 * 10^{-6}} + \underset{\begin{pmatrix} z = 43{,}83 \\ p = 0{,}0000 \end{pmatrix}}{0{,}6746\ \sigma_{t-1}^2} + \underset{\begin{pmatrix} z = 12{,}98 \\ p = 0{,}0000 \end{pmatrix}}{0{,}3552\ \varepsilon_{t-1}^2} + \varepsilon_t \quad ,$$

with generalized error distribution.

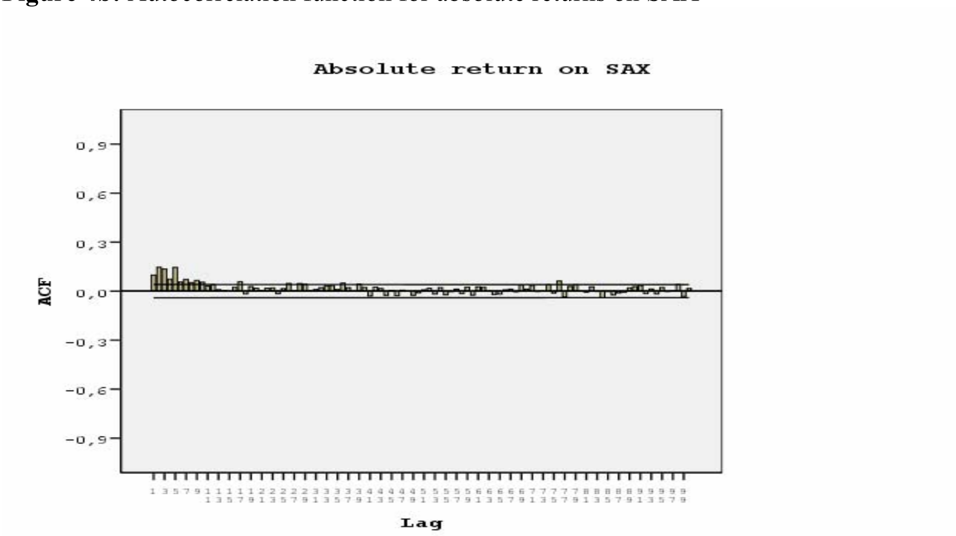
For the beginning, we check the successfulness of the LAVE method against the usual GARCH(1,1) model by comparing standardization of absolute returns. Namely, for all analyzed time series we observe highly persistent autocorrelation function of absolute daily returns, as presented in Figure 4a for SBI20. This statement holds for all analyzed time series, except for SAX, where only few first lags are slightly significant (see Figure 4b). Exceptionally low stock turnover on Bratislava Stock Exchange, in absolute as well as in relative sense (only 0,2 % of stock turnover in total turnover – as opposed to 40,8 % at Ljubljana Stock Exchange, 43,0 % at Warsaw Stock Exchange, 62,5 % at Budapest Stock Exchange, 66,1 % at Prague Stock Exchange, 65,5 % at London Stock Exchange and 83,4 % at German Stock Exchange – Source: own calculations based on FESE data), very probably causes greater randomness of stock trading.

Figure 4a: Autocorrelation function for absolute returns on SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

Figure 4b: Autocorrelation function for absolute returns on SAX

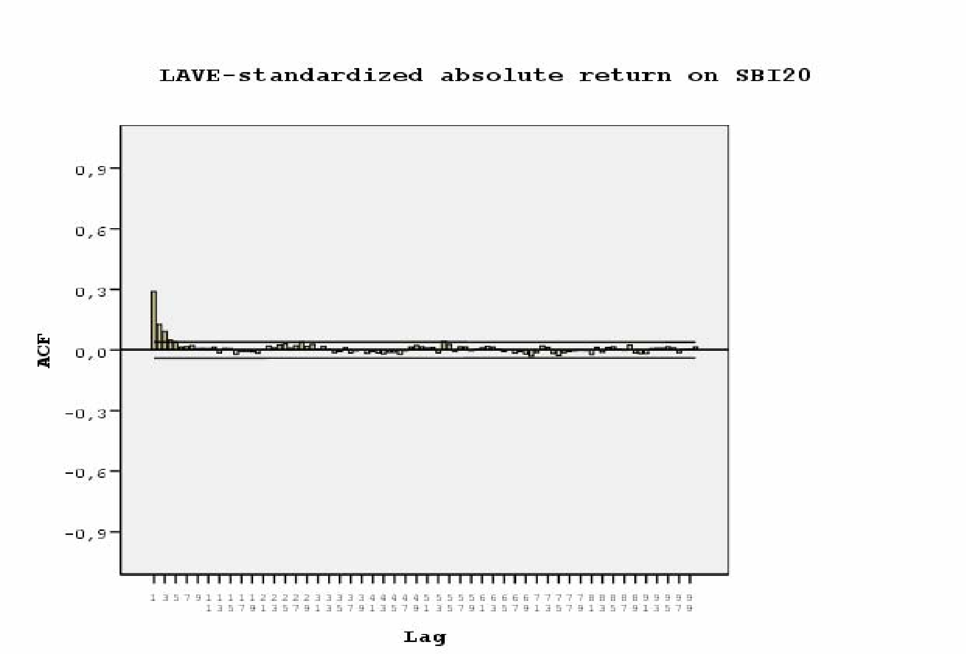


Source: Own calculations based on the ISI Emerging Markets data

In order to determine which model is more appropriate for dealing with this issue, we compare the autocorrelations of standardized absolute returns, i.e.

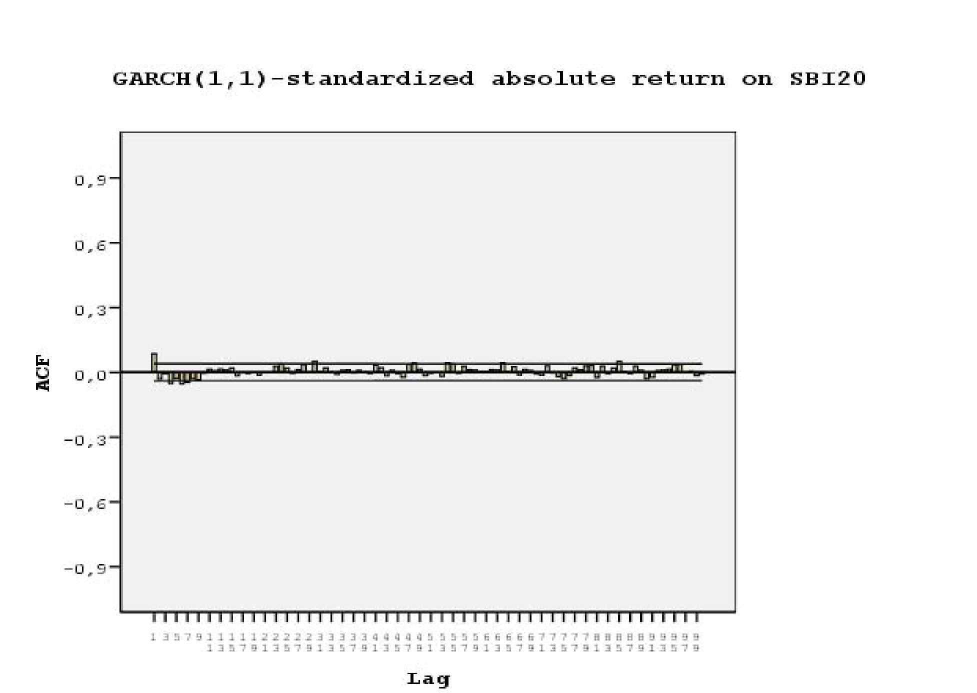
returns divided by the estimated standard deviation (volatility). The following figures represent autocorrelations of absolute daily returns on SBI20, standardized by conditional standard deviations as estimated by LAVE procedure (Figure 5a), and autocorrelations of absolute daily returns on SBI20, standardized by conditional standard deviations as estimated by GARCH(1,1) model (Figure 5b).

Figure 5a: Autocorrelation function for absolute daily returns on SBI20, standardized by LAVE



Source: Own calculations based on the data from Ljubljana Stock Exchange

Figure 5b: Autocorrelation function for absolute daily returns on SBI20, standardized by GARCH(1,1)



Source: Own calculations based on the data from Ljubljana Stock Exchange

We observe that the autocorrelation function of standardized absolute returns is not significant any more, except for the first lag by GARCH(1,1) standardization and the first three lags by LAVE standardization. This leads us to the conclusion that explaining the data by LAVE approach is quite satisfactory but not as successful as with GARCH(1,1). The same conclusion can be drawn also for the rest of the analyzed indexes, even for SAX.

To compare the two methods also via quality of forecasting, we check this other aspect in a similar way as Mercurio and Spokoiny (2004) did. Since volatility is a hidden process, it can be observed only together with the multiplicative error. According to (1), it holds that $E({R_{t+1}}^2 | \Im_t) = {\sigma_{t+1}}^2$, and therefore the *QFC* (*Quality of the Forecast Criterion*) can be defined as

$$QFC = \frac{1}{T - t_0 - 1} \sum_{t=t_0}^{T} \left| R_{t+1}{}^2 - \hat{\sigma}_{t+1|t}{}^2 \right|^p .$$

The two authors were interested in a robust criterion, not too sensitive to the outliers, so they used the value of $p = 0.5$ instead of more common $p = 2$. In their research the results for $p = 2$ were in favor of GARCH(1,1).

If we, too, set $p = 0.5$, and observe the two models' goodness of fit for the whole analyzed period, we get the results as presented in Table 3:

Table 3: Relative fitting performance, period 1996-2005, $p = 0.5$

Index	QFC **LAVE** / QFC **GARCH(1,1)**
PX	1,006
BUX	1,054
WIG20	1,002
SAX	0,987
SBI20	1,048
FTSE100	1,033
DAX	1,037

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

The results show that in our case for $p = 0.5$ the GARCH(1,1) method is superior for all index daily return time series, except for SAX. However, in our opinion, outliers are of a crucial importance, as far as modeling volatility in stock or index returns is concerned. It is very likely that they are not a consequence of a measurement error, but simply the fact. Properly detecting the timing and possibly the amplitude of a price-shock is a huge advantage at stock trading, and therefore the method shouldn't be too robust to outliers. Thus, we use $p = 2$ instead of $p = 0.5$ and get the following results:

Table 4: Relative fitting performance, period 1996-2005, $p = 2$

Index	$QFC_{\text{LAVE}} / QFC_{\text{GARCH(1,1)}}$
PX	0,998
BUX	0,971
WIG20	1,006
SAX	0,974
SBI20	0,971
FTSE100	1,025
DAX	1,034

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

Here, the results are a little bit more in favor of LAVE, but they are not very convincing, either.

However, since we are more interested in volatility forecasting than fitting, we also check how the two compared models behave in the forecasting sense. For this purpose we estimate the GARCH(1,1) model on the first nine observed years only and then use the obtained model to produce forecasts through the whole last year. The forecasting model representation, based on data from 1996 till 2004, for SBI20 is:

$$\sigma_t^2 = 2{,}4661*10^{-6} + 0{,}7019\ \sigma_{t-1}^2 + 0{,}3285\ \varepsilon_{t-1}^2 + \varepsilon_t$$
$$\begin{pmatrix} z = 4{,}67 \\ p = 0{,}0000 \end{pmatrix} \quad \begin{pmatrix} z = 50{,}23 \\ p = 0{,}0000 \end{pmatrix} \quad \begin{pmatrix} z = 13{,}38 \\ p = 0{,}0000 \end{pmatrix} \quad ,$$

again with generalized error distribution.

The LAVE method estimates and forecasts the volatility simultaneously.

Table 5: Relative forecasting performance for the year 2005 (GARCH(1,1) model is estimated on the basis of data from 1996 till 2004), $p = 2$

Index	$QFC_{\text{LAVE}} / QFC_{\text{GARCH(1,1)}}$
PX	0,925
BUX	0,909
WIG20	0,945
SAX	1,042
SBI20	1,202
FTSE100	0,973
DAX	0,893

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

Now, the conclusion regarding volatility modeling is not a straightforward one. From Table 3 we can observe that in our case, for $p = 0.5$, the LAVE approach is not appropriate at all. Table 4 includes estimates for $p = 2$ and shows somewhat more favorable situation for the alternative method, and Table 5 contains similar results.

Therefore, even under different assumptions, we cannot reliably claim that our findings support those of Mercurio and Spokoiny (2004) about LAVE method outperforming GARCH(1,1) in general.

4.3. Level modeling

Nevertheless, because of its simplicity and no need of assumptions, we checked whether the LAVE method can be successfully used in level modeling as well.

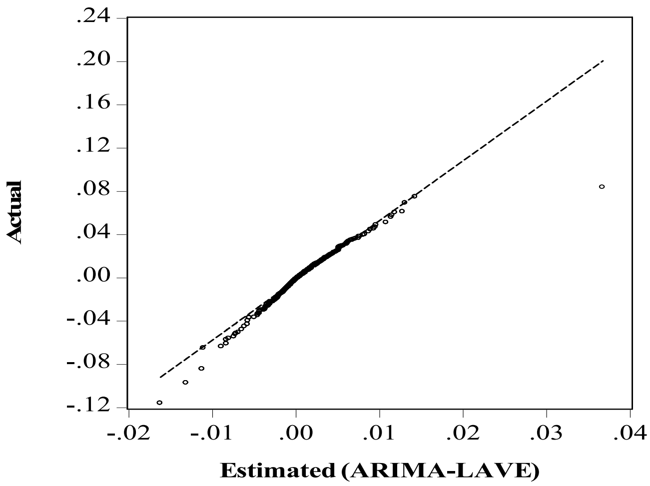
Like with volatility modeling, we try to determine the goodness of fit first. For this purpose we estimate the two compared models on the basis of all observed data, i.e. from 1996 till 2005. The presented ARIMA(0,0,1)-LAVE model specification is again calculated for SBI20:

$$R_t = \underset{\begin{pmatrix} t = 8{,}14 \\ p = 0{,}0000 \end{pmatrix}}{0{,}1613}\varepsilon_{t-1} + \underset{\begin{pmatrix} t = 2{,}54 \\ p = 0{,}0113 \end{pmatrix}}{0{,}0495} LAVE_t + \varepsilon_t \quad , \tag{4}$$

where again $LAVE = \sqrt{\tilde{\theta}_I} = \sqrt{\frac{1}{|I|}\sum_{i \in I} |R_i|^2}$ with I being the corresponding interval of homogeneity.

In Figure 6 the quantile-quantile (Q-Q) plot for comparison of the distribution of the estimated against the distribution of the actual values for daily returns on SBI20 is presented.

Figure 6: Quantile-quantile plot – comparison of the distribution of the ARIMA-LAVE-estimated values against the distribution of the actual values for daily returns on SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

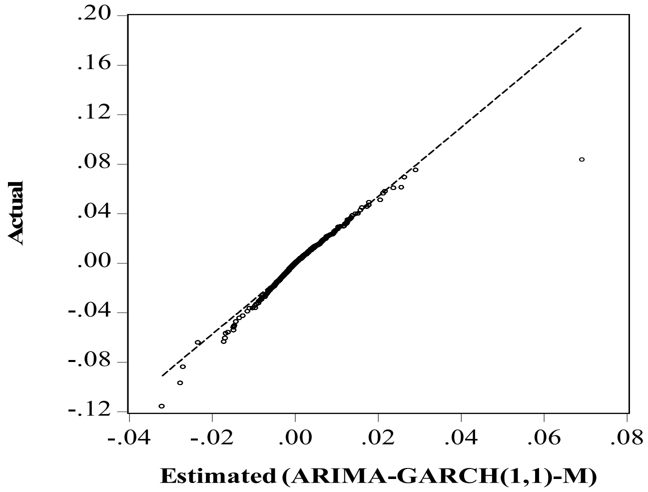
From Figure 6 we observe that the ARIMA-LAVE model is quite satisfactory at capturing the characteristics of return distribution. Except from a part of the left tail and an outlier at the far right end of distribution, the points form a straight line, indicating the similarity of the two compared distributions.

Comparison of ARIMA-LAVE with ARIMA-GARCH(1,1)-M model

To compare the LAVE level-modeling performance with a more common and recognized approach, we do the return estimation also by the ARIMA(1,0,0)-GARCH(1,1)-M model (*GARCH-in-Mean* form as proposed by Engle, Lilien and Robins, 1987) (again, estimated on the data from the whole analyzed period 1996-2005):

$$\begin{aligned} R_t &= 0{,}0689\ \sigma_t + 0{,}3171\ R_{t-1} \\ &\quad \begin{pmatrix} z = 2{,}26 \\ p = 0{,}0238 \end{pmatrix} \begin{pmatrix} z = 16{,}82 \\ p = 0{,}0000 \end{pmatrix} \\ \sigma_t^2 &= 2{,}2488 * 10^{-6} + 0{,}7178\ \sigma_{t-1}^2 + 0{,}2990\ \varepsilon_{t-1}^2 + \varepsilon_t \\ &\quad \begin{pmatrix} z = 5{,}96 \\ p = 0{,}0000 \end{pmatrix} \begin{pmatrix} z = 52{,}41 \\ p = 0{,}0000 \end{pmatrix} \begin{pmatrix} z = 12{,}48 \\ p = 0{,}0000 \end{pmatrix} \end{aligned} \qquad (5)$$

Figure 7: Quantile-quantile plot – comparison of the distribution of the ARIMA-GARCH-M-estimated values against the distribution of the actual values for daily returns on SBI20



Source: Own calculations based on the data from Ljubljana Stock Exchange

Figure 7 presents an almost identical Q-Q plot as Figure 6.

From the Q-Q point of view we cannot decide which of the two methods serves better in estimating the original time series. We can only see that, regarding the similarity of the actual and estimated distributions, both methods are satisfactory. The same is observed from Q-Q plots for PX and WIG20 as well, whereas for BUX and SAX the tails of distributions are not modeled properly, neither with LAVE nor with GARCH correction. For FTSE100 and DAX, too, the two compared models do not fit the data best, as far as distribution is concerned – the actual distribution is namely even more leptokurtic than the modeled one. But, taking Q-Q plots into account, unlike the rest of the analyzed indexes, FTSE100 and DAX are modeled somewhat better with ARIMA-GARCH(1,1)-M than with ARIMA-LAVE approach.

As for analytical comparison, it is difficult to decide which error measurement to use. Armstrong and Collopy (1992) show that there is no error measure that would serve best in general. From the most widely used measures they argue in favor of MAPE (*Mean Absolute Percentage Error*) over MSE (*Mean Square Error*). MSE is shown to be a poor protection against outliers. However, MAPE is relevant for ratio-scaled data (with meaningful zero), which is not the case here. Besides, the outliers cause a problem only if we compare the successfulness of a method across many time series. We, on the contrary, only compare the methods separately for each time series. Armstrong and Collopy (1992) also stress, that their study ignores large errors that are sometimes the primary concern, and that in such case the MSE might be appropriate. To avoid the dilemma, we introduce a compromise that was briefly mentioned (but not used or described) already by Armstrong and Collopy (1992) – we calculate the MSPE (*Mean Square Percentage Error*):

$$MSPE = \left(\sum_{t=T+1}^{T+h} \left(\frac{\hat{y}_t - y_t}{y_t} \right)^2 \right) \Big/ h ,$$

where $\hat{y}_t$ are forecasts, y_t are actual values, T is number of observations in the sample, and $(T+1\,,\,T+h)$ is a time interval of the forecasts made on the basis of the first T observations.

In Table 6 the MSPE values are stated for the whole 10-year period for models such as (3) and (4) for SBI20.

Table 6: MSPE, period 1996-2005 – the lowest value is bolded

Index	ARIMA-LAVE	ARIMA-GARCH(1,1)-M
PX	**1,73**	2,54
BUX	**68,24**	142,52
WIG20	**5,31**	10,12
SAX	4,84	**1,03**
SBI20	**70,45**	228,35
FTSE100	**1,11**	2,31
DAX	**1,16**	2,91

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

As far as goodness of fit, measured by MSPE, is concerned, it is obvious that the original time series is more successfully fitted by the proposed ARIMA-LAVE model than by the GARCH model. The only exception is SAX time series.

Again, we are more interested in forecasting than fitting, so we compare the two modeling approaches also from the forecasting point of view. We estimate both volatility ARIMA modifications on the first nine observed years and then use the obtained model to forecast the daily return through the whole last year. The forecasting model representations, based on data from 1996 till 2004, for SBI20 are thus:

ARIMA(0,0,1)-LAVE:

$$R_t = \underset{\begin{pmatrix} t = 7{,}55 \\ p = 0{,}0000 \end{pmatrix}}{0{,}1579\varepsilon_{t-1}} + \underset{\begin{pmatrix} t = 2{,}43 \\ p = 0{,}0153 \end{pmatrix}}{0{,}0499 LAVE_t} + \varepsilon_t$$

and

ARIMA(0,0,1)-GARCH(1,1)-M:

$$R_t = 0{,}0675\ \sigma_t + 0{,}3282\ \varepsilon_{t-1}$$
$$\begin{pmatrix} z = 2{,}22 \\ p = 0{,}0267 \end{pmatrix} \begin{pmatrix} z = 15{,}24 \\ p = 0{,}0000 \end{pmatrix}$$

$$\sigma_t^2 = 1{,}3848 * 10^{-6} + 0{,}7708\ \sigma_{t-1}^2 + 0{,}2616\ \varepsilon_{t-1}^2 + \varepsilon_t \ .$$
$$\begin{pmatrix} z = 8{,}18 \\ p = 0{,}0000 \end{pmatrix} \begin{pmatrix} z = 203{,}66 \\ p = 0{,}0000 \end{pmatrix} \begin{pmatrix} z = 31{,}27 \\ p = 0{,}0000 \end{pmatrix}$$

The MSPE for the forecasts are presented in Table 7.

Table 7: MSPE for the year 2005 (the two forecasting models are estimated on the basis of data from 1996 till 2004) – the lowest value is bolded

Index	ARIMA-LAVE	ARIMA- GARCH(1,1)
PX	**1,32**	1,95
BUX	**4,43**	12,27
WIG20	3,46	**3,20**
SAX	**0,94**	1,02
SBI20	**4,95**	13,45
FTSE100	**1,05**	2,85
DAX	**0,98**	1,34

Sources: Own calculations based on the data from Ljubljana Stock Exchange, ISI Emerging Markets, [http://finance.yahoo.com/q/hp?S=%5EFTSE] and [http://finance.yahoo.com/q/hp?S=%5EDAXI]

The ARIMA-LAVE approach is, for six of the seven analyzed stock index time series – this time the exception is WIG20 – apparently more successful also as a forecasting tool.

5. DISCUSSION AND CONCLUDING REMARKS

Mercurio and Spokoiny (2004) proposed an alternative approach to volatility estimation, called LAVE (*Locally Adaptive Volatility Estimate*), that, in their research, outperformed the renowned GARCH(1,1) model. Besides, LAVE is a simple non-parametric method and therefore we don't need to bother about the model specification. The model is estimated promptly and the programming algorithm is not too involving and is easy to understand.

We applied the LAVE method on a different type of data to check whether LAVE can be successfully used for volatility estimation of other sorts of time series as well. Instead of exchange rates for nine developed countries we analyzed daily returns of five stock indexes from emerging markets of Central Europe (and two additional from the developed markets).

The results of volatility modeling are not as convincing as expected. The differences (with regard to the original empirical application) may arise from the different type of data. As far as comparison of the two time series from developed stock markets to the other five time series from emerging markets is concerned, no distinction could be drawn between the two samples. (However, the SAX stock index daily return time series was many times an exception, and this is probably due to very low stock turnover in Bratislava.) Therefore the reason for different behavior of the method on our data set as compared to the one of Mercurio and Spokoiny (2004) is very likely not the state of development of the financial markets. The distinctive point is probably that the stock index returns are much more volatile than the exchange rates that were analyzed by the two authors.

Nevertheless, we propose another use of LAVE calculations. We set up a model for level estimation by including the estimated conditional volatility as an additional explanatory variable in the basic ARIMA model. When compared to ARIMA-GARCH-M model, the new model proves to be more successful.

Since there are many possibilities of optimizations on both sides (changing grid step, smoothing parameter and power transformation for LAVE; using different modifications and moving (rolling) estimate for GARCH), none of the two methods is to be dismissed. LAVE approach is easily understood, more intuitive, and doesn't require parametric assumptions, while GARCH models are well-known and included in statistical packages. But of course, it can always be argued also about which ARIMA model representation to use and about which error measure to rely on.

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URL: [http://www.ljse.si/]