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MEASURING THE RISK EFFICIENCY IN INDIAN COMMERCIAL BANKING - A DEA APPROACH

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ABSTRACT

This study proposes data envelopment analysis models to identify and asses risk in Indian commercial banks. Risk is believed to surface due to external and internal factors, where the former cannot be controlled and the later can be controlled fully or partially by the bank management. 63 commercial banks comprising public, private and foreign sectors exposed to common frontier production function are considered for performance evaluation. The empirical results are interesting.

KEY WORDS: Data Envelopment Analysis, Endogenous Risk, Exogenous Risk, Technical Efficiency, Scale Efficiency, Commercial Bank.

JEL Classification: G21, G32

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INTRODUCTION

To start with Indian commercial banking was dominated by private ownership; profit and return to investments were its performance indicators. This was the scenario prior to 1969. Subsequently, commercial banks were nationalized adding to their list additional objectives of optimizing social benefit and geographical expansion to meet the growing needs of people. Globalization opened gates to increased competition by the entry of foreign banks. The changes that are taking world wide continued to give shocks to the banking system which resulted in an expansion of banking services both in range, volume and non-performing assets.

Gauging efficiency of commercial banks is an important issue to bank management and the policy maker. Before this task is initiated a commercial bank has to be modeled appropriately to meet the needs and objectives of the analyst.

To model a commercial bank two approaches followed mostly are the intermediation and production approaches. Under the intermediation approach financial institutions are viewed intermediate funds between depositors and borrowers (Piyu, Y., 1992). Banking business has to satisfy both the users and suppliers of bank funds. The intermediation approach is also known as the asset approach. In production approach a commercial bank's resources produce services to the customers (Berg et.al, 1991; Berg et.al. 1993; Parson et.al, 1993; Shaffnit et.al, 1997). The basic difference is that in production approach deposits are treated as output, where as it has input status in intermediate approach.

The user cost approach or profit approach models a commercial bank differently. According to user cost approach a financial product is an input or output on the basis of its net contribution to bank's revenue. If returns on a financial product exceed the opportunity cost then it is treated as output otherwise input (Berger et.al., 1993; Thompson et.al, 1997). In the name of loan losses a proxy to NPA, Brocket et.al (1997) included variants of loan losses in both input and output list. 'Provision for loan losses' appeared in input list, allowances for loan losses in the output list. The later output variable was defined as, Valuation Reserve-Loan losses. Sueyoshi (2001) attempted to measure financial performance and to group the banks using DEA discriminent analysis. To measure the risk factor a ratio named as 'Bank Loan loss Ratio' was used in the discriminent analysis. In the context of Indian Commercial Banks' performance measurement adequate representation is not given to risk

as measured by NPA (Bhattacharya et.al, 1996; Asish Saha, T.S.Ravisankar (2000).

Performance of banks and bank branches was studied by a number of analysts, but unfortunately there is no general agreement of choice of technology in terms of inputs and outputs.(Bhattacharya et.al, 1996; Parson et.al, 1993; Hevary Tulkers 1993; Berger et.al, 1993; English et.al,1993; Chaffai 1997; Brocket et.al 1997; De Yong Reber 1997; Mester Loreta 1997; Humphrey David 1993; Berg et.al, 1991; Kumbhakar et.al, 1998).

DATA

The present study models a commercial bank in production approach perspective. The study accommodates non-performing assets as an undesirable output which can serve as input, invoking user cost approach. Therefore, this work is a blend of production and user cost approaches.

For the inputs we use (1) Number of employees and (2) Fixed Assets. Desirable outputs are (1) Deposits (2) Loans and Advances (3) Investments (4) Non-interest income and undesirable output is Non-Performing Assets (NPA). (Fare and Grosskopf, 2004; Scheel, 2001; Seiford and Zhu, 2002) The data are secondary arise from the balance sheets submitted to the Reserve Bank of India by the commercial banks.

Adding too many inputs and outputs to DEA list of variables in the presence of too small a number of decision making units leads to loss of discriminatory power of DEA, since in this case a large proportion of DMUs will surface with 100% efficiency score (Hughs and Yaisawarng, 2004). Thus, an analyst shall be objective oriented and parsimonious while inputs and outputs are listed to confront with DEA. The present study considers 63 Public, Private and Foreign sector banks.

DEA MODELS

Charnes, Cooper and Rhodes (1978) introduced a multiplier problem to measure input technical efficiency of a decision making unit in a competitive environment where similar inputs are employed to produce similar outputs. The specification was a fractional programming problem. By employing Charnes-Cooper transformation it can be transformed into a linear programming problem. But this problem implicitly assumes scale efficient environment. Banker, Charnes and Cooper (1984) made the necessary modification to model

variable returns to scale. The dual formulations of these problems are called data envelopment analysis problems. The linear programming problems pursued in the study are given in Appendix-I.

NON PERFORMING ASSETS AS UNDESIRABLE OUTPUT:

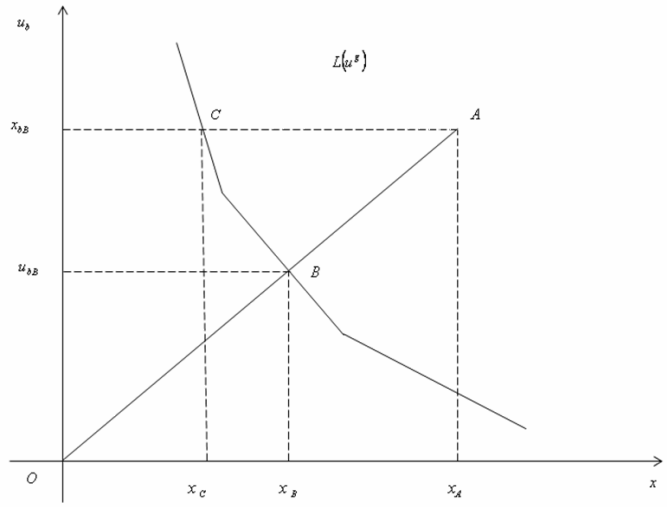

Figure (1)

In the figure above $L(u^g)$ is the input level set consisting of all input (x) and undesirable output (u_b) pairs which can lead to produce u^g, good output. Input and undesirable output are measured respectively along the horizontal and vertical axes. If efforts are needed to control input and NPA losses, the targets occur at B which is reached by radial reduction of $(x_A, \mathrm{u}_{\mathrm{bA}})$. Consequently, the input target is $x_B \left(= \lambda^{\text{rcite}}_{\text{CRTS}}\ x_A\right)$. If risk is left uncontrolled, then the decision making unit is set target to operate at C, whose input is $x_C \left(= \lambda^{\text{rucite}}_{\text{CRTS}}\ x_A\right)$. $x_C \leq x_B$

The input target at C dominates the input target at B. We always have

$$\lambda^{\text{rcite}}_{\text{CRTS}} \geq \lambda^{\text{rucite}}_{\text{CRTS}}$$

$$\Leftrightarrow \lambda^{\text{rcite}}_{\text{CRTS}} x_0 \geq \lambda^{\text{rucite}}_{\text{CRTS}} x_0$$

where, CRTS refers to Constant Returns to Scale.

This implies that there is a tradeoff between input saving and NPA saving. $(\lambda_{\text{CRTS}}^{\text{rcite}} - \lambda_{\text{CRTS}}^{\text{rucite}})\, x_0$ measures additional input targets over risk controlled input targets. In value terms, if input prices are known, the additional input cost saving is measured by $(\lambda_{\text{CRTS}}^{\text{rcite}} - \lambda_{\text{CRTS}}^{\text{rucite}})\, p_0^T\, x_0$, where p_0 is observed price vector of $\text{DMU}_0\left(x_0, u_{\text{b0}}, u_0\right)$. In risk controlled environment not only input reduction but also NPA (= u_{b}) reduction is sought. The value of recovered NPA is measured by the following expression:

$$(1 - \lambda_{\text{CRTS}}^{\text{rcite}})\, u_{b0} = \left(1 - \lambda^{\text{icre}}\right) u_{b0}$$

NPA –CHOICE OF ENVIRONMENT

We assume a trade off between inputs cost and NPA. In a commercial bank if potential inputs cost gains dominate in some sense NPA gains the choice of environment is allocate resources and effort to reduce inputs to their optimum level without bothering NPA. This situation occurs for banks operating at relatively low levels of NPA. Inputs are reduced physically and their productivity is increased simultaneously to achieve risk uncontrolled input technical efficiency. For such commercial banks the in built risk controlling mechanism is strong enough that there is no need to allocate resources to further control NPA. Such allocation leads to only marginal gains of NPA at the expense of sizable inputs cost.

A commercial bank may choose one of risk exogenous or endogenous environments, making use of the following decision rule:

$$\left(\lambda_{CRTS}^{rcite} - \lambda_{CRTS}^{rucite}\right) p_0 x_0 \begin{cases} < & \Rightarrow \text{Endogenous risk environmen t} \\ \geq \left(1 - \lambda_{CRTS}^{rcite}\right) u_{b0} & \Rightarrow \text{Exogenous risk environmen t} \end{cases}$$

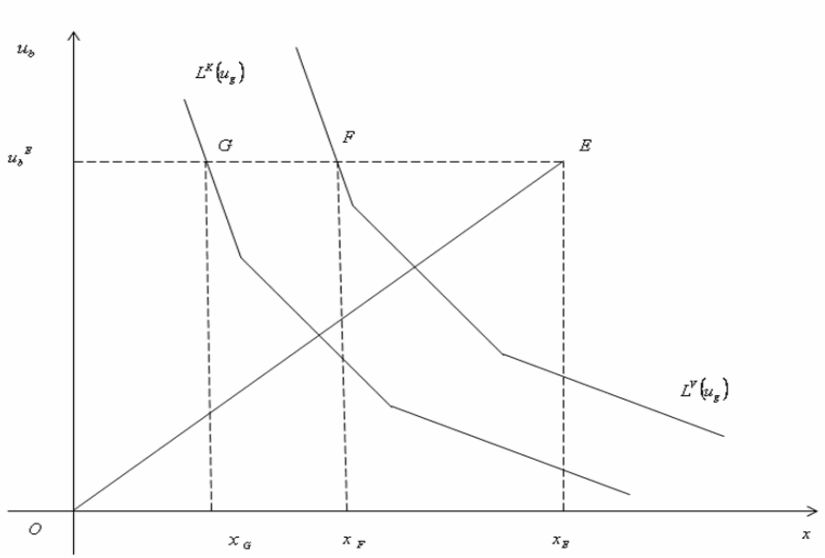

Figure (2)

The input level sets $L^v(u_g)$ and $L^k(u_g)$ are formulated under the hypotheses of variable and constant returns to scale respectively. u_g refers to a scalar valued desirable (good) output.

$$L^v(u_g) \subseteq L^k(u_g)$$

Risk factor as measured by u_b^E is held constant and input x_E is reduced to reach the points F (x_E, u_b^E) and G (x_G, u_b^E) of isoquant $L^v(u_g)$ and isoquant $L^k(u_g)$ respectively.

$$x_F = \lambda_{VRTS}^{rucite} \; x_E$$

$$x_G = \lambda_{CRTS}^{rucite} \; x_E$$

where VRTS refers to Variable Returns to scale.

The ratio $\frac{\lambda_{CRTS}^{rucite}}{\lambda_{VRTS}^{rucite}} = \lambda^{rucse}$, measures risk uncontrolled input scale efficiency. Risk uncontrolled input technical efficiency can be decomposed into the product of risk exogenous input pure technical and scale efficiency.

$$\lambda_{CRTS}^{rucite} = \left(\frac{\lambda_{CRTS}^{rucite}}{\lambda_{VRTS}^{rucite}} \right) \left(\lambda_{VRTS}^{rucite} \right)$$

$$= \underbrace{(\lambda_{VRTS}^{rucite})}_{\text{pure technical efficiency}} \quad \underbrace{(\lambda^{rucise})}_{\text{scale efficiency}}$$

Similarly the ratio, $\lambda^{rcse} = \left(\frac{\lambda_{CRTS}^{rcite}}{\lambda_{VRTS}^{rcite}} \right)$ measures input scale efficiency when risk also is reduced at the same rate as inputs. The risk controlled input technical efficiency can be decomposed into the product of input scale and pure technical efficiency.

$$\lambda_{CRTS}^{rcite} = \left(\frac{\lambda_{CRTS}^{rcite}}{\lambda_{VRTS}^{rcite}} \right) \left(\lambda_{VRTS}^{rcite} \right)$$

$$= \underbrace{(\lambda^{rcise})}_{scale\ \text{efficiency}} \quad \underbrace{(\lambda^{rcite}_{VRTS})}_{pure\ \text{technical efficiency}}$$

EXOGENOUS RISK - INPUT CONTROL

Most of the studies concerned with Indian commercial banks performance ignored the importance of NPA, which is an indicator of risk. If the risk factor is ignored the technical efficiency of a commercial bank is under stated. Let us consider two environments, one which ignores risk and another that minimizes risk in some sense. The most ideal environment for a commercial bank is zero-NPA and no scale inefficiency which prevails if returns to scale are constant. The input technical efficiency under risk free and CRTS environment (λ^{rfite}_{CRTS}) can be evaluated solving a linear programming problem.

In the risk free environment under constant returns to scale λ^{rfite}_{CRTS} is input technical efficiency. If risk is imposed as a constraint in its exogenous form, input technical efficiency is λ^{rucite}_{CRTS}. Since $\lambda^{rfite}_{CRTS} \leq \lambda^{rucite}_{CRTS}$, we have, $0 \leq \frac{\lambda^{rfite}}{\lambda^{rucite}} \leq 1$ and this ratio measures effective inputs that are accounted for by exogenous risk inducement.

ENDOGENOUS RISK- INPUTS CONTROL: (RISK CONTROL - ENDOGENOUS INPUTS)

Every commercial bank possesses a risk control system to control endogenous risk. Unlike the previous environment where risk is exogenous it is hypothesized that the conversional inputs and NPA can be radially controlled at the same rate as measured by λ^{rcite}_{CRTS}.

The ratio $\frac{\lambda_{crts}^{rfite}}{\lambda_{crts}^{rcite}}$ measures effective inputs attributed to endogenous risk. Input target values diminish at the rate of $\frac{\left(\lambda_{CRTS}^{rcite} - \lambda_{CRTS}^{rucite}\right)}{\overline{\lambda}_{CRTS}}$ and the associated risk targets will increase at the rate of, $\frac{\left(1 - \lambda_{CRTS}^{rcite}\right)}{\overline{\overline{\lambda}}_{CRTS}}$.

Input Control – Endogenous Risk, and Risk Control-Endogenous inputs environments are the same in this study.

The overall input technical efficiency can be decomposed as,

$$\lambda_{crts}^{rfite} = \underbrace{\left(\frac{\lambda_{crts}^{rfite}}{\lambda_{crts}^{rucite}}\right)\left(\frac{\lambda_{crts}^{rucite}}{\lambda_{vrts}^{rucite}}\right)\left(\lambda_{vrts}^{rucite}\right)}_{Exogenous\ \ Risk} = \underbrace{\left(\frac{\lambda_{crts}^{rfite}}{\lambda_{crts}^{rcite}}\right)\left(\frac{\lambda_{crts}^{rcite}}{\lambda_{vrts}^{rcite}}\right)\left(\lambda_{vrts}^{rcite}\right)}_{Endogenous\ \ Risk}$$

EXOGENOUS INPUTS- RISK CONTROL

If inputs are uncontrolled risk targets are enhanced, possible by strengthening the in built risk controlling system to carefully evaluate the consequence of a loan or investment. In this case what ever funds are allocated to increase the productivity of inputs are directed to risk management.

In input uncontrolled risk environment the risk targets dominate those of input controlled risk targets. NPA gains are greater where risk is measured in input uncontrolled environment. However, there are input losses due to increased risk control activity.

ENDOGENOUS INPUTS-RISK CONTROL (ENDOGENOUS RISK – INPUTS CONTROL)

If risk is exogenous to a commercial bank, arises, for example, due to political intervention, the welfare schemes introduced by the Govt., Reserve Bank of India monitory policies, competition effect attributed to external competitive environment from other commercial banks and financial institutions, conventional input losses can be estimated with knowledge from λ_{CRTS}^{rucite}. In risk

environment which is endogenous, risk can be controlled by further strengthening its internal risk control system.

$$\lambda_{CRTS}^{rucite} \leq \lambda_{CRTS}^{rcite}$$

When risk is left free so that it is exogenous to the commercial bank, all the resources and efforts are diverted to further reduce inputs physically and increase their productivity. Input targets when risk is exogenous dominate risk endogenous input targets.

EMPIRICAL ANALYSIS

We evaluate the performance of 63 commercial banks operating on Indian soil. Unlike traditional decomposition, input technical efficiency measure in risk free constant returns to scale environment is decomposed into the product of risk, scale and pure input technical efficiency. Risk in bank's competitive environment is measured by non-performing assets (NPA). It is the only way to account for risk in Indian banking business. The most ideal environment that a commercial bank would like to function is no-NPA and constant returns to scale. We shall name this risk free scale efficient environment. Input losses experienced by public and private sector banks in this environment are significantly larger than those experienced by foreign sector banks.

If risk is treated as non-discretionary input, it influences the bank environment exogenously, under scale efficient environment, the risk uncontrolled input technical efficiency, λ_{crts}^{ruite} can be derived solving a linear programming problem.

Table (1)
RISK FREE, CONSTANT RETURNS TO SCALE INPUT TECHNICAL EFFICIENCY

Sectors	Minimum	Maximum	Mean	SD	Coefficient of Variation (%)
Public Sector Banks	0.1807	0.7833	0.3174	0.1132	35.66
Private Sector Banks	0.0584	0.6919	0.2965	0.1508	50.86
Foreign Sector Banks	0.1208	1.0000	0.6845	0.2831	41.36

Table (2)
RISK UNCONTROLLED INPUT TECHNICAL EFFICIENCY IN CONSTANT RETURNS TO SCALE ENVIRONMENT

Sector	Minimum	Maximum	Mean	SD	Coefficient of Variation (%)
Public Sector Banks	0.4520	1.0000	0.7908	0.1797	22.72
Private Sector Banks	0.0584	1.0000	0.5464	0.2688	49.19
Foreign Sector Banks	0.3739	1.0000	0.8507	0.2039	23.93

In risk uncontrolled scale efficient environment the risk constraint dramatically increases input technical efficiency. State Bank of India, the largest commercial bank experienced 70 percent input losses in risk free environment, experienced no input losses in exogenous risk environment, caused by factors like political intervention, implementation of Govt., welfare schemes and so on . Failure to control risk leads to input losses more in private sector banks than public and foreign sector banks.

The risk controlled input technical efficiency in scale efficient environment treats risk endogenous, which can be controlled strengthening internal risk control mechanism, for example, by administering controls on the size of the loan, careful evaluation of the credibility of the borrower and the collateral security, investments leading to the best opportunity costs, motivating employees to make them feel their belongedness and spreading risk are some ways of controlling risk in commercial bank business. The private sector banks experienced huge input losses compared to the public and foreign sector banks. In private sector of commercial banks 40 percent of inputs are lost on the average due to input technical inefficiency measured in endogenous risk and scale efficient environment. On the average foreign and public sector banks experienced 13 and 19 percent of input losses respectively due to input technical inefficiency. The built in risk control system is weakest for private sector banks. In this sector irrational leading has lead to failure of a number of private sector banks.

Thus, due to input technical inefficiency the private sector banks experienced significantly more input losses than the public and foreign sector banks. These banks should strive hard to reach input saving and endogenous risk reducing bench marks, possible if efforts simultaneously strengthen input technical efficiency and the inbuilt risk control mechanism.

EXOGENOUS INPUTS – RISK CONTROL

Letting the conventional inputs exogenously fixed, the potential NPA recovery can be assessed solving a linear programming problem.

The resultant efficiency estimate is called ‘ecological efficiency’, which provides a lower bound to risk efficiency.

$$\lambda_{CRTS}^{iucre} \leq \lambda_{CRTS}^{icre} \leq 1$$

Table (3)
RISK CONTROLLED INPUT TECHNICAL EFFICIENCY IN CONSTANT RETURNS TO SCALE ENVIRONMENT (Inputs Controlled Risk Efficiency)

Sector	Minimum	Maximum	Mean	SD	Coefficient of Variation (%)
Public Sector Banks	0.4673	1.0000	0.8138	0.1671	20.53
Private Sector Banks	0.2339	1.0000	0.5995	0.2339	39.02
Foreign Sector Banks	0.3739	1.0000	0.8727	0.1849	21.19

Table (4)
Inputs Uncontrolled Risk Efficiency in Constant Returns to Scale Environment:

Sector	Minimum	Maximum	Mean	SD	Coefficient of Variation (%)
Public Sector Banks	0.0001	1.0000	0.4325	0.4027	93.12
Private Sector Banks	0.0000	1.0000	0.1411	0.2854	202.24
Foreign Sector Banks	0.0008	1.0000	0.6151	0.4238	68.90

There are five ecological efficient banks among the public sector banks, State Bank of India, State Bank of Patiala, Andhra Bank, IDBI Ltd. and Oriental Bank of Commerce. Given that inputs are exogenous these banks cannot reduce their NPA implying that the existing risk is completely attributed to exogenous factors. Their internal risk control system is strong enough that there is no need to expend resources to further strengthen it.

The structural ecological efficiency of the public sector banks is 0.4325 inferring that under inputs exogenous hypothesis 53 percent of observed NPA would not have taken place if the public sector banks had adequate strength to control endogenous risk.

Among private sector banks, the Axis Bank and ICICI Bank are the only two commercial banks enjoyed eco-efficiency. In choice of environment analysis these banks were advised to function under risk exogenous input control environment. If Catholin Syrian Bank, Dhana Lakshmi Bank, Lord Krishna Bank, Sangli Bank strengthen their endogenous risk control system NPA can be recovered completely. The mean eco-efficiency of private commercial banking sector is 0.1411 implying that about 86 percent of the NPA can be recovered if greater commitment is shown and efforts are made. The risk faced by banks of Private sector is more endogenous than exogenous.

Among foreign sector banks ABN Amro Bank, Bank of Tokyo-Mitsubhishi UFJ, China trust commercial Bank, City Bank, Deutsche Bank and JB Morgan Chase Bank have attained 100% eco-efficiency score. For these banks NPA accumulation is not due to endogenous factors. In choice of environment analysis also these banks are advised to choose to compete in risk exogenous environment. The structural eco-efficiency of foreign sector banks is 0.6151. If inputs are exogenous strengthening the interval risk control system this banking sector would have prevented 38 percent of NPA from accruing.

Input scale efficiency is measured by the ratio, $\frac{\lambda_{crts}^{rcite}}{\lambda_{vrts}^{rcite}}$ and $\frac{\lambda_{crts}^{rucite}}{\lambda_{vrts}^{rucite}}$ in endogenous and exogenous risk situations respectively.

Under exogenous risk hypothesis marginal input losses (6%) are experienced by the public and foreign sector banks due to scale inefficiency where as 14 percent of inputs are lost in private sector.

TABLE (5)
INPUT SCALE EFFICIENCY

Sector	Risk Uncontrolled Environment (Average)	Risk Controlled Environment (Average)
Public	0.9406	0.8471
Private	0.8579	0.6102
Foreign	0.9391	0.8774

In risk controlled environment 39 percent of inputs are freely disposed in private sector due to scale inefficiency; however, in public and foreign sector these losses are 15 and 12 percents respectively.

TABLE (6)
INPUT PURE TECHNICAL EFFICIENCY

Sector	Exogenous Risk Hypothesis	Endogenous Risk Hypothesis
Public	0.8398	0.9587
Private	0.6451	0.9829
Foreign	0.9004	0.9943

Under exogenous risk and variable return to scale environment, private sector banks experienced significantly more input losses compared to public and private sector banks. The input pure technical efficiency is obtained removing the influence of exogenous risk, and scale inefficiency from overall input technical efficiency measured in no-NPA, scale efficient environment. All the three sectors experienced input losses only marginally in endogenous risk environment.

Among public sector banks risk exogenous environment is experienced by six banks, viz., State Bank of India, State Bank of Patiala, Andhra Bank, Bank of India, IDBI Ltd., and Oriental Bank of Commerce. For these banks the existing risk control mechanism in strong enough, and there is no need of expending

any more resources to control risk. The remaining inefficient commercial banks require to choose endogenous risk environment.

79 percent of the public sector commercial banks are required to reduce their inputs and NPA.

Among 23 private sector banks 21 banks should control their inputs as well as risk. The ICICI bank appears to be efficient in risk exogenous and endogenous environment. 91 percent of commercial banks of private sector should perform in endogenous risk environment, strengthening their inbuilt risk control system and efforts are needed to increase the productivity of inputs.

Among 12 foreign sector banks 6 banks are efficient in both the risky environments. Four banks are required to reduce their inputs and non-performing assets.

CONCLUSIONS

This study decomposes multiplicatively input technical efficiency measured in risk free scale efficient environment into risk, scale and pure technical efficiency. A decision rule is proposed for the choice of environment. The risk faced by a commercial bank arises from forces operating from outside and within leading to exogenous and endogenous risk respectively. The former inefficiency emanates from factors that a commercial bank cannot control, and the later due to the weakness of the inbuilt risk control system.

The study compares 63 commercial banks comprising public, private and foreign sector banks against a common non-parametric production frontier. The empirical results reveal that exogenous risk is menace more to the public sector banks than foreign and private sector banks.

The in built risk control system is equally strong for public and foreign sector banks, 0.8138 for public and 0.8727 for foreign sector banks measured in terms of effective input utilization. Irrational loan advances, investments are prominent more in private sector than public sector banks measured in terms of risk efficiency. Consequently, the private sector banks should strengthen their internal risk control system. Due to scale inefficiency more inputs are lost in private sector banks than public sector banks. The Foreign sector banks are well ahead in their performance in all respects than public and private sector banks.

APPENDIX-I

DEA Models:

$$\lambda_{CRTS}^{rfite} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , i = 1,2,\ldots,m\right\}$$

$$\lambda_{CRTS}^{rucite} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , \sum_{j=1}^{n} \lambda_j \, u_{bj} \le u_{b0}, i = 1,2,\ldots,m; r = 1,2,....,s\right\}$$

$$\lambda_{VRTS}^{rucite} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , \sum_{j=1}^{n} \lambda_j \, u_{bj} \le u_{b0}, \sum_{j=1}^{n} \lambda_j = 1, \; i = 1,2,\ldots,m; r = 1,2,....,s\right\}$$

$$\lambda_{CRTS}^{rcite} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , \sum_{j=1}^{n} \lambda_j \, u_{bj} \le u_{b0}, i = 1,2,\ldots,m; r = 1,2,....,s\right\}$$

$$\lambda_{VRTS}^{rcite} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , \sum_{j=1}^{n} \lambda_j \, u_{bj} \le u_{b0}, \sum_{j=1}^{n} \lambda_j = 1, \; i = 1,2,\ldots,m; r = 1,2,....,s\right\}$$

$$\lambda_{CRTS}^{iucre} = Min\left\{\lambda : \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \lambda \, x_{i0} \, , \sum_{j=1}^{n} \lambda_j \, u_{rj} \ge u_{r0} \, , \sum_{j=1}^{n} \lambda_j \, u_{bj} \le \lambda u_{b0}, i = 1,2,\ldots,m; r = 1,2,....,s\right\}$$

$$\lambda_{VRTS}^{icre} = \lambda_{VRTS}^{ricte}$$

APPENDIX-II

PUBLIC SECTOR BANKS:

S.NO	Bank Name	λ_{CRTS}^{rfite}	λ_{CRTS}^{rucite}	λ_{VRTS}^{rucite}	λ_{CRTS}^{rcite}	λ_{VRTS}^{rcite}	λ^{rucse}	λ^{rcse}	λ_{CRTS}^{iucre}
1	State Bank of India	0.2992	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	State Bank Bikaner & Jaipur	0.3408	0.8849	0.9435	0.9037	0.9958	0.9379	0.9075	0.6583
3	State bank of Hyderabad	0.3439	0.8837	0.8905	0.9097	0.9543	0.9924	0.9533	0.6086
4	State Bank of Indore	0.3809	0.8469	0.9651	0.8510	0.9979	0.8775	0.8528	0.3662
5	State Bank of Mysore	0.2946	0.7238	0.7260	0.7725	0.9666	0.9970	0.7992	0.2644
6	State bank of Patiala	0.4284	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	State bank of Saurashtra	0.2218	0.4773	0.4944	0.4950	0.9591	0.9654	0.5161	0.0001
8	State Bank of Travancore	0.3679	0.8764	1.0044	0.8783	1.0000	0.8726	0.8783	0.6117
9	Allahabad Bank	0.2651	0.5593	0.6164	0.5962	0.9082	0.9074	0.6565	0.0522
10	Andhra Bank	0.3463	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	Bank of Baroda	0.3107	0.9386	0.9589	0.9488	0.9589	0.9788	0.9895	0.6893
12	Bank of India	0.3201	0.9960	1.0000	0.9964	1.0000	0.9960	0.9964	0.9687
13	Bank of Maharashtra	0.2689	0.6861	0.7930	0.7618	0.9763	0.8652	0.7803	0.0688
14	Canara Bank	0.2783	0.6269	0.6310	0.6612	0.8181	0.9935	0.8082	0.1664
15	Central bank of India	0.2060	0.6772	0.7627	0.7038	0.9268	0.8879	0.7594	0.0004
16	Corporation Bank	0.3752	0.8978	0.8993	0.9192	0.9728	0.9983	0.9449	0.9978
17	Dena Bank	0.2430	0.5360	0.7739	0.5454	0.9760	0.6926	0.5588	0.0173
18	IDBI Ltd.	0.7833	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	Indian Bank	0.2012	0.7119	0.7233	0.7556	0.8951	0.9842	0.8442	0.0402
20	Indian Overseas Bank	0.3006	0.8673	0.8744	0.8856	0.9310	0.9919	0.9512	0.3324

21	Oriental Bank of Commerce	0.4353	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	Punjab & Sind Bank	0.1807	0.4520	0.4543	0.4673	0.9442	0.9949	0.4949	0.0001
23	Punjab national Bank	0.2677	0.8601	0.8697	0.8707	0.8767	0.9890	0.9932	0.0254
24	Syndicate Bank	0.2912	0.7692	0.7692	0.8229	0.9313	1.0000	0.8836	0.2381
25	UCO Bank	0.2730	0.6683	1.0000	0.7266	1.0000	0.6683	0.7266	0.0339
26	Union bank of India	0.3163	0.7941	0.8355	0.8329	0.9462	0.9504	0.8803	0.1961
27	United Bank of India	0.1880	0.4635	0.5830	0.5360	0.9257	0.7950	0.5790	0.0114
28	Vijaya Bank	0.3588	0.9446	0.9450	0.9463	0.9819	0.9996	0.9637	0.7621

FOREIGN SECTOR BANKS:

S.NO	**Bank Name**	λ_{CRTS}^{rfite}	λ_{CRTS}^{rucite}	λ_{VRTS}^{rucite}	λ_{CRTS}^{rcite}	λ_{VRTS}^{rcite}	λ^{rucse}	λ^{rcse}	λ_{CRTS}^{iucre}
29	ABN Amro bank	0.6895	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	Abu Dhabi Commercial Bank	0.9239	0.9239	1.0000	0.9239	1.0000	0.9239	0.9239	0.2539
31	American Express Bank	0.1208	0.5921	0.5930	0.7690	0.9905	0.9985	0.7764	0.4674
32	Bank of Bahrain & Kuwait	0.3739	0.3739	0.4821	0.3739	0.9979	0.7756	0.3747	0.0008
33	Bank of Ceylon	0.8064	0.8064	1.0000	0.8064	1.0000	0.8064	0.8064	0.0469
34	Bank of Tokyo-Mitsubishi UFJ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	Chinatrust Commercial Bank	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

36	Citi Bank	0.7371	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
37	Deutsche Bank	0.6148	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
38	Hong Kong & Shanghai Banking Corporation	0.4722	0.8009	0.8024	0.8266	0.9520	0.9981	0.8683	0.4543
39	JB Morgan Chase bank	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
40	Standard Chartered Bank	0.4755	0.7107	0.9270	0.7723	0.9909	0.7667	0.7794	0.1585

PRIVATE SECTOR BANKS:

S.NO	**Bank Name**	λ_{CRTS}^{rfite}	λ_{CRTS}^{rucite}	λ_{VRTS}^{rucite}	λ_{CRTS}^{rcite}	λ_{VRTS}^{rcite}	λ^{rucse}	λ^{rcse}	λ_{CRTS}^{iucre}
41	Axis Bank	0.5112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
42	Bank of Rajasthan	0.2238	0.5351	0.5355	0.5632	0.9680	0.9993	0.5818	0.0067
43	Catholin Syrian Bank	0.1644	0.3254	0.4828	0.3919	0.9905	0.6740	0.3957	0.0000
44	Centurion Bank of Punjab	0.1159	0.2515	0.3286	0.3891	0.9303	0.7654	0.4183	0.0025
45	Citi Union Bank	0.2725	0.4994	0.5441	0.5641	0.9922	0.9178	0.5685	0.0013
46	Development Credit bank	0.2082	0.3646	0.4271	0.4358	0.9889	0.8537	0.4407	0.0052
47	Dhanalakshmi bank	0.1893	0.3041	0.3660	0.4082	0.9920	0.8309	0.4115	0.0000
48	Federal Bank	0.3419	0.7654	0.7655	0.7745	0.9726	0.9999	0.7963	0.1531
49	HDFC Bank	0.2874	0.8221	0.8351	0.8514	0.9008	0.9844	0.9452	0.2406
50	ICICI Bank	0.6919	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

51	Induslnd Bank	0.5864	0.9127	1.0000	0.9164	1.0000	0.9127	0.9164	0.0496
52	Ing Vysys bank	0.2961	0.4889	0.4944	0.5056	0.9572	0.9889	0.5282	0.0156
53	Jammu & Kashmir Bank	0.3601	0.8196	0.9619	0.8262	0.9966	0.8521	0.8290	0.2575
54	Karnataka Bank	0.3180	0.7031	0.8354	0.7200	0.9909	0.8416	0.7266	0.1033
55	Karur Vysya Bank	0.3007	0.5790	0.5793	0.6086	0.9760	0.9995	0.6236	0.0067
56	Kotak Mahindra bank	0.2920	0.5697	1.0000	0.7157	1.0000	0.5697	0.7157	0.2865
57	Lakshmi Vilas bank	0.2953	0.5581	0.7730	0.6160	0.9967	0.7220	0.6180	0.0104
58	Lord krishna Bank	0.1389	0.1666	0.8597	0.3457	0.9990	0.1938	0.3460	0.0000
59	Ratnakar bank	0.1355	0.1355	0.1547	0.2449	0.9952	0.8759	0.2461	0.0000
60	Sangli Bank	0.0584	0.0584	0.0611	0.1326	0.9837	0.9558	0.1348	0.0000
61	SBI Comm.& Intl Bnak	0.4509	0.4509	0.4966	0.4509	0.9976	0.9080	0.4520	0.0011
62	South Indian Bank	0.3063	0.7056	0.7621	0.7266	0.9879	0.9259	0.7355	0.0811
63	Tamilnad Mercantile Bank	0.2748	0.5516	0.5740	0.6003	0.9901	0.9610	0.6063	0.0251

NOTE: 1. $\lambda^{rucse} = \lambda^{rucite}_{CRTS} \big/ \lambda^{rucite}_{VRTS}$

2. $\lambda^{rcse} = \lambda^{rcite}_{CRTS} \big/ \lambda^{rcite}_{VRTS}$

Appendix: III

Public sector Banks – Choice of environment:

S.No	Bank Name	$(\lambda_{crts}^{rcite} - \lambda_{crts}^{rcite})\, p_0\ \ x_0$	$(1-\lambda_{crts}^{rcite})\, u_{b0}$	Risk Environment
1	State Bank of India	0.0000	0.0000	Exogenous
2	State Bank Bikaner & Jaipur	1012.2400	2145.5640	Endogenous
3	State bank of Hyderabad	1034.3548	553.5390	Exogenous
4	State Bank of Indore	101.6887	2369.9940	Endogenous
5	State Bank of Mysore	1827.0677	1703.5200	Exogenous
6	State bank of Patiala	0.0000	0.0000	Exogenous
7	State bank of Saurashtra	428.6321	3914.2550	Endogenous
8	State Bank of Travancore	86.3796	3256.9359	Endogenous
9	Allahabad Bank	1253.4520	17774.8700	Endogenous
10	Andhra Bank	0.0000	0.0000	Exogenous
11	Bank of Baroda	1919.0158	2568.5500	Endogenous
12	Bank of India	73.3200	227.5300	Endogenous
13	Bank of Maharashtra	3973.3560	6607.1916	Endogenous
14	Canara Bank	6666.9905	31405.7400	Exogenous
15	Central bank of India	3567.5574	266006.3600	Endogenous
16	Corporation Bank	942.7751	1146.7944	Endogenous
17	Dena Bank	410.9930	16720.1880	Endogenous

18	IDBI Ltd.	0.0000	0.0000	Exogenous
19	Indian Bank	4264.8507	2475.6312	Exogenous
20	Indian Overseas Bank	2025.4879	2949.5752	Endogenous
21	Oriental Bank of Commerce	0.0000	0.0000	Exogenous
22	Punjab & Sind Bank	637.4334	4103.9208	Endogenous
23	Punjab national Bank	2675.3393	9382.2666	Endogenous
24	Syndicate Bank	5918.6600	6924.7871	Endogenous
25	UCO Bank	5512.0000	27505.6804	Endogenous
26	Union bank of India	3797.2148	10046.3862	Endogenous
27	United Bank of India	4736.2575	15318.0000	Exogenous
28	Vijaya Bank	77.9774	773.0652	Endogenous

Foreign Sector Banks – Choice of Environment:

S.No	**Bank Name**	$(\lambda_{crts}^{rcite} - \lambda_{crts}^{rcite})\, p_0\ x_0$	$(1-\lambda_{crts}^{rcite})\, u_{b0}$	**Risk Environment**
29	ABN Amro bank	0.0000	0.0000	Exogenous
30	Abu Dhabi Commercial Bank	0.0000	9.9340	Endogenous
31	American Express Bank	2596.9663	248.3600	Exogenous
32	Bank of Bahrain & Kuwait	0.0000	11.8900	Endogenous
33	Bank of Ceylon	0.0000	117.7088	Endogenous

34	Bank of Tokyo-Mitsubishi UFJ	0.0000	0.0000	Exogenous
35	Chinatrust Commercial Bank	0.0000	0.0000	Exogenous
36	Citi Bank	0.0000	0.0000	Exogenous
37	Deutsche Bank	0.0000	0.0000	Exogenous
38	Hong Kong & Shanghai Banking Corporation	1949.5655	1709.3772	Exogenous
39	JB Morgan Chase bank	0.0000	0.0000	Exogenous
40	Standard Chartered Bank	4467.0950	9834.0000	Endogenous

Private Sector Banks – Choice of Environment:

S.No	**Bank Name**	$(\lambda_{crts}^{rcite} - \lambda_{crts}^{rcite})\, p_0\ \ x_0$	$(1-\lambda_{crts}^{rcite})\, u_{b0}$	**Risk Environment**
41	Axis Bank	0.0000	0.0000	Exogenous
42	Bank of Rajasthan	515.1244	588.3700	Endogenous
43	Catholin Syrian Bank	669.0392	3629.1400	Endogenous
44	Centurion Bank of Punjab	4033.3766	8644.2350	Endogenous
45	Citi Union Bank	338.6765	1580.1375	Endogenous
46	Development Credit bank	609.6558	2462.1688	Endogenous
47	Dhanalakshmi Bank	525.1241	1907.9632	Endogenous
48	Federal Bank	269.3224	1466.8775	Endogenous

49	HDFC Bank	2651.7444	3014.9454	Endogenous
50	ICICI Bank	0.0000	0.0000	Exogenous
51	Induslnd Bank	47.8351	2288.5500	Endogenous
52	Ing Vysys bank	462.9951	5637.1488	Endogenous
53	Jammu & Kashmir Bank	156.3426	3370.6772	Endogenous
54	Karnataka Bank	237.1712	3249.1200	Endogenous
55	Karur Vysya Bank	320.2927	625.0658	Endogenous
56	Kotak Mahindra bank	5764.3297	6163.6240	Endogenous
57	Lakshmi Vilas bank	364.9382	2186.8800	Exogenous
58	Lord krishna Bank	448.5846	2909.6721	Exogenous
59	Ratnakar bank	269.5365	770.6202	Endogenous
60	Sangli Bank	460.4995	1258.5974	Endogenous
61	SBI Comm.& Intl Bnak	0.0000	3.0510	Endogenous
62	South Indian Bank	322.6524	2127.3254	Endogenous
63	Tamilnad Mercantile Bank	480.6802	1590.4063	Endogenous

APPENDIX: IV

Data in lakhs

S.No	Bank Name	No. of Employees	Fixed Assets	Deposits	Advances	Investments	Non-Interest income	NPA
1	State Bank of India	185388	281886	43552109	33733649	14914888	617556.11	525772
2	State Bank Bikaner & Jaipur	11752	14225	2848049	2052622	868367	40373.74	22280
3	State Bank of Hyderabad	12800	24222	4150267	2810925	1391915	50033.34	6130
4	State Bank of Indore	6517	10455	1997649	1535138	599244	23300.46	15906
5	State Bank of Mysore	9604	13338	2202235	1646553	698975	37311.27	7488
6	State Bank of Patiala	11329	16351	3918363	2876976	1235766	36070.12	23841
7	State Bank of Saurashtra	7148	19643	1580488	1108114	501100	12061.99	7751
8	State Bank of Travancore	11607	16036	3098401	2478628	956169	24315.6	26762
9	Allahabad Bank	20379	105634	5954366	4129004	1874606	41274.88	44019
10	Andhra Bank	13831	19234	4145402	2788907	1430073	48016.42	4725
11	Bank of Baroda	38086	108880	12491598	8362087	3494364	130263.03	50167
12	Bank of India	41511	78930	11988173	8493590	3549275	174213.5	63203
13	Bank of Maharashtra	13893	21490	3391934	2291939	1129840	29257.12	27738
14	Canara Bank	46359	286135	14238144	9850569	4522553	160982.2	92697
15	Central Bank of India	39055	76727	8277628	5179547	2774190	53014.61	87800
16	Corporation Bank	11880	28103	4235689	2994965	1441750	63791.99	14193
17	Dena Bank	10120	44187	2768991	1830339	923504	42458.34	36480
18	IDBI Ltd.	7482	277835	4335403	6247082	2567532	111108.07	72193

19	Indian Bank	20892	55118	4709091	2905812	2087772	79169.6	10213
20	Indian Overseas Bank	23861	51067	6874042	4706028	2397449	45241.26	25783
21	Oriental Bank of Commerce	14730	38268	6399597	4413847	1980835	67282.01	21566
22	Punjab & Sind Bank	9325	25301	1931875	1173751	669308	24378.95	7704
23	Punjab national Bank	57316	100983	13985968	9659652	4518983	110447.31	72562
24	Syndicate Bank	24360	77154	7863357	5167044	2523402	73207.43	39101
25	UCO Bank	24773	66669	6486001	4698891	1952487	48661.52	100606
26	Union Bank of India	27536	82500	8518023	6238643	2798178	73928.07	60122
27	United Bank of India	16793	60522	3716666	2215632	1460182	35963.3	33300
28	Vijaya Bank	10765	18617	3760449	2422355	1201841	31344.54	14396
29	ABN Amro Bank	3549	12404	1599830	1838755	640667	86930.38	2128
30	Abu Dhabi Commercial Bank	40	672	47379	20344	14356	1649.69	128
31	American Express Bank	1982	5648	266411	159318	195896	39870.39	1231
32	Bank of Bahrain & Kuwait	76	587	36430	17090	10894	455.68	19
33	Bank of Ceylon	33	13	8649	4094	3087	333.14	608
34	Bank of Tokyo-Mitsubishi UFJ	142	1237	96049	158857	52238	5481.23	113
35	Chinatrust Commercial Bank	21	26	10287	11590	4115	136.49	32
36	Citi Bank	5194	79797	3787501	3286110	1602114	159923.55	33610
37	Deutsche Bank	1040	13030	697838	494506	620354	77769.33	42
38	Hongkong & Shanghai Banking Corporation	6564	70216	3482465	2314168	1413083	144066.33	9858
39	JB Morgan Chase Bank	130	202	166656	79925	436669	23511.83	1738
40	Standard Chartered Bank	7321	87727	3417405	3010379	1190229	151252.67	43190
41	Axis Bank	9980	67320	5878560	3687648	2689717	120141.87	26633

42	Bank of Rajasthan	3908	13888	1081593	570402	364069	13669.61	1367
43	Catholin Syrian Bank	2791	5986	474859	301264	155329	3763.45	5968
44	Centurion Bank of Punjab	14458	33731	1486372	1122135	461496	50273.16	14150
45	Citi Union Bank	1871	3922	469933	332923	130700	6113.87	3625
46	Development Credit bank	1809	8144	441520	265852	184663	10787.78	4364
47	Dhanalakshmi Bank	1385	4957	308796	183950	86519	3241.07	3224
48	Federal Bank	6029	18609	2158444	1489910	703266	31362.39	6505
49	HDFC Bank	21477	96667	6829794	4694478	3056480	167873.52	20289
50	ICICI Bank	33321	392341	23051019	19586560	9125783	685869.6	199204
51	Induslnd Bank	2613	36958	1764481	1108420	589166	26577.49	27375
52	Ing Vysya Bank	4982	39596	1541858	1197617	452780	20829.17	11402
53	Jammu & Kashmir Bank	6829	18345	2519428	1707994	739218	16614.98	19394
54	Karnataka Bank	4456	10682	1403744	955268	504816	18169.23	11604
55	Karur Vysya Bank	3286	9669	934031	704048	287395	13184	1597
56	Kotak Mahindra Bank	5437	14108	1100009	1092406	686196	37640.07	21680
57	Lakshmi Vilas Bank	1926	3549	501987	361271	130930	4952.76	5695
58	Lord krishna Bank	1163	2061	187252	101782	72398	1398.72	4447
59	Ratnakar Bank	553	1622	87638	53052	31583	504.51	1020
60	Sangli Bank	1770	8124	132589	20507	79647	614.56	1451
61	SBI Comm.& Intl Bnak	97	4675	48785	32950	12567	433.99	61
62	South Indian Bank	3868	8959	1223921	791892	343013	11468.17	7781
63	Tamilnad Mercantile Bank	2227	4928	601988	404672	231639	8929.49	3979

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