



# **ACID DEPOSITION IN URBAN AREAS: AN EQUILIBRIUM APPROACH**

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## **Abstract**

This paper presents an equilibrium model of a market for differentiated products that is offered for studying the effects of acid deposition on residential areas. Analytical solutions are computed for all the endogenous variables. All the cross equation restrictions of the model are specified. The model is offered for empirical work and has nice aggregation properties. To investigate whether there are acid deposition effects on urban areas, the model looks at the rental and owner's occupied housing markets. Markets are modeled under the assumption of rational expectations.

**Keywords:** hedonic models, differentiated goods, structural analysis, acid deposition

**JEL Classification:** D01, D04, D90, D59, D40, D49

## **Introduction**

A new approach to the theory of individual choices is based on the realization that some goods or factors of production are not homogeneous and can differ in numerous characteristics. Houthakker (1952) pioneered this approach to the problem of quality variation and to the theory of consumer behavior. Becker (1965), Lancaster (1966), Muth (1966), and Rosen (1974) adopted and extended this new approach.

The hedonic equilibrium price equation is an equilibrium that results from the interactions of the suppliers and demanders of a differentiated product and it contains information on the underlying preferences and technologies. Rosen (1974) studied

briefly the possibility to extract this information for a situation that has a differentiated product with only one characteristic and realized that the calculations required even for this simplified case are quite complex. Therefore, he proposed a methodology for estimating the demands and supplies of characteristics in a second stage rather than using the hedonic equation directly.

To extract information from the hedonic equilibrium price equation, the analytical (closed-form or structural as it is else called) approach was first pursued by Epple (1984). Epple generalizes Tinbergen's (1959) model to treat a commodity with an arbitrary number of attributes and introduces both endogenous demand and supply. However, these models have several restrictive features. Namely, the cross partial derivative of the utility function is zero, the marginal utility with respect to the numeraire good is constant (hence the income elasticity of demand for the product is zero), the variance-covariance matrices of the exogenously given distributions have to be diagonal or satisfy other restrictions, the number of consumer characteristics equals the number of product characteristics, and the price equation parameters are not unique.

Giannias (1990), (1989 a & b) assume that a linear function maps physical characteristics into a scalar quality index and that economic agents care only about the quality of the commodity that they purchase and not for the individual characteristics of the product. While this is a strong assumption, this quality index technology allows to impose weaker a priori restrictions in other respects. The result is a class of models with closed-form solutions that does not have the restrictive features enumerated above. This paper presents closed form-solutions to a dynamic hedonic model where markets are modeled under the assumption of rational expectations and the product characteristics affect directly the utility function and not indirectly through a quality index equation. This model presents a framework for assessing the effects of acid deposition.

Burning of coal and oil by industrial plants, particularly power-generating plants, and the automobiles cause acid deposition. The fossil fuel plants create sulfur dioxide emissions and the automobile creates nitrogen oxide emissions. These emissions concentrate in the moisture of the atmosphere and come down in rain, snow or fog. For example, Pennsylvania as well as Ohio in the USA are two states that receive a lot of acid rainfall.

Acid precipitation causes damages to materials, fisheries, forests, agriculture, and other aspects of society, including public health. Urban residents perceive some of this damage and their happiness depend (directly or indirectly) on acid deposition. For example, damages on materials that affect gross rental housing prices, health effects, effects on recreational activities, and aesthetic effects that determine locational decisions. The latter assumes that consumers would like to live in (and

therefore, they would pay more for) a house that is located close to parks, rivers, forests, etc if there is not an acid deposition problem. However, as the acid deposition increases and as damages on trees, rivers, etc start to occur, the consumers would like to move into houses that are located further from parks, rivers, forests, etc because consumers tend to dislike living close to dead or polluted rivers, forests, etc.

For policy applications, it is important to find out (theoretically and empirically) the consumer willingness to pay for less acid deposition. This paper presents a model that is able to infer willingness to pay for decreases in acid deposition from an analysis of the housing market. This is justified if housing-locational decisions of consumers depend on acid deposition because of real acid deposition effects that matter to them. This paper presents an equilibrium model for an analysis of the effects of acid deposition. The model suggests methods of testing whether acid deposition affects housing-location decisions. If housing-location decisions within urban areas depend on acid deposition, it can be concluded that there are acid deposition damages that are perceived by households in residential areas and the equilibrium willingness to pay for decreases in acid deposition can be computed for given estimation results.

The theory studies the behavior of heterogeneous buyers (different preferences and income) who consume two goods, a differentiated good (housing) and the numeraire good. To derive analytical solutions for the consumer demand for the differentiated product, I i) take a second order approximation of the utility function around the mean values and ii) use a hedonic rental housing equation that has some nice properties for assessing acid deposition effects. Then the restrictions among all the parameters of the model are specified. These restrictions reveal the relationship between the parameters of the hedonic price equation and the parameters of consumer preferences and supply distribution, and can be used to test the internal consistency of the theory.

In most studies of markets for differentiated products that I am aware of i) there is an inconsistency between the theoretical model and the empirical part since the theoretical model is written in terms of annual rental prices, although housing values are used for the estimation, e.g., Harrison and Rubinfeld (1978), and ii) when census tract data is used for the estimation, there is an aggregation problem that is never discussed or treated. In this paper i) and ii) are treated explicitly.

A nice feature of the method developed in this paper for obtaining analytical solutions is that, unlike previous work in modelling hedonic equilibrium models, e.g., Tinbergen (1959) and Epple (1984), preferences do not have to be separable, marginal utilities do not have to be constant, and the demand for housing characteristics does not have to have a zero income elasticity. Moreover, to obtain analytical solutions for the demand for housing characteristics that have all the nice

properties that are needed for the study of the acid deposition problem, there is no need to introduce into the model a quality index equation of the kind that is employed in Giannias (1989, a & b), (1990), (1991).

## **The Economic Model.**

### **Economic Agents - Housing Contracts.**

The economy consists of buyers who consume two goods  $h$  and  $x$ , where  $h$  is a  $(1 \times n)$  vector of attributes that describes the differentiated good housing, and  $x$  is the numeraire good. The  $n$ -th element of  $h$  is the proximity to parks, rivers, etc. The supply for  $x$  is infinitely elastic and consumers are assumed to use one unit of housing.

Each consumer is assumed to live and work in a given urban area (e.g., a certain metropolitan area). Given his income and preferences a consumer makes a housing-locational decision within the given area. It is assumed that there is no across urban areas migration. A consumer's utility function depends on the acid deposition of the urban area he lives in,  $r$ . The acid deposition is uniform within an urban area, that is,  $r$  is the same in all parts of it. The acid deposition,  $r$ , is assumed to be a random variable with a distribution that is a public information and stays the same in all time periods. Future realizations of  $r$  are assumed to be unknown.

It is assumed that there are two groups of consumers, Group A and Group B. Group A consists of consumers who do not mind i) searching to find the current  $r$  value, and ii) moving from one house to another. Group B consists of consumers who dislike i) and ii) strongly.

The consumers of Group A are going into the housing market every year to rent a house. The consumers of Group B have decided to make a long term commitment that allows them to live into the same house the rest of their life.

On the other side of the economy there are competitive firms that supply housing services to consumers. Competitive firms take the housing price distribution as given and build the houses that maximize their profit. It is assumed that, once a house is built, it is there forever. So, a firm builds the type of houses that maximize profit and offers those housing services into the market for an infinite number of time periods. It is assumed that there are only two types of housing contracts traded in the market, the A- and B-types.

The buyer (supplier) of an A-type contract agrees to use (to supply) the services of an  $h$ -type house for one year and pay (receive) in return  $P(h,r)$ , where  $P(h,r)$  is the

market price for the one year services of an h-type house when r is the realization of the acid deposition variable.

The buyer (supplier) of a B-type contract agrees to use (to supply) the services of an h-type house for an infinite number of years, i.e., the rest of his life, and pay (receive) in return  $v(h,r)$  each year.

Note that for an equilibrium it has to be satisfied:  $P(h,r) = v(h,r)$  for all h and r, so that the firms are indifferent to what consumers' group they sell housing services. Finally, it is assumed that neither consumers nor firms can break a contract.

### **Equilibrium.**

Let the total number of consumers in the urban area considered, N, be an exogenously given constant for all time periods, and  $N_t(h)$  be the total number of people who have decided to live in a h-type house at time t in the urban area considered. Obviously, integrating  $N_t(h)$  over  $(a_t, b_t)$  we obtain N, where all the houses that are available in the given area at t are in the (vector) interval  $(a_t, b_t)$ .

For an equilibrium at t, it has to be satisfied:  $N_t(h) = T_t(h)$  for all h in  $(a_t, b_t)$ , where  $T_t(h)$  is the t-period offer for h-type housing services in the given urban area. The equilibrium condition implies  $T_t = N$  for all t, where  $T_t$  is the total number of houses that are available at time t.

Now suppose that at  $t=0$  the suppliers decide to build the  $T_0(h)$  distribution of houses and that the economy reaches an equilibrium. So,  $T_0(h) = N_0(h)$  for all h in  $(a_0, b_0)$ .

It is assumed that every house has a positive value. The latter implies that new house will not be built in future periods,  $t = 1, 2, , \dots$ . That implies  $(a_t, b_t) = (a_0, b_0)$  for all t.

Let  $s_0(h)$  be the number of h-type houses that are available for the satisfaction of an A-housing contract in the given urban area at  $t = 0$ , and  $S_0(h)$  be the number of h-type houses that are employed for the satisfaction of a B-housing contract in all time periods. Note that the definitions imply:  $T_0(h) = s_0(h) + S_0(h)$ . Given the structure of the economy,  $s_0(h)$  and  $S_0(h)$  are going to be the distributions of housing characteristics that are available to buyers of an A- and B-housing contract, respectively, in the given urban area at  $t = 1, 2, 3, 4, \dots$

An equilibrium requires that the following conditions are satisfied:

$$d_t(h) = s_0(h) \text{ for all h and t, and}$$

$$D_0(h) = S_0(h) \text{ for all } h,$$

where  $d_t(h)$  is the number of people who want to rent an  $h$ -house in the given urban area at  $t$ , and  $D_0(h)$  is the number of people who demand a B-type contract on an  $h$ -house at  $t = 0$ . For understanding of the notation note that:  $d_t(h) + D_0(h) = N_t(h)$  for  $t = 0, 1, 2, 3, 4, \dots$

In the next sections,  $h(A)$  and  $h(B)$  will denote the means of the distributions  $s_0(h)$  and  $S_0(h)$  respectively.

### **Consumer Preferences.**

Preferences are described by utility functions. The utility of an  $a$ -type consumer in one year from the consumption of an  $(h,x)$ -bundle is assumed to be of the following quadratic form:

$$U(h,x,r,a) = k(r) + (k_0 + a k_1 + i k_2 r) (h - h^*)' + 0.5 (h - h^*) k_3 (h - h^*)' \\ + k_4 x + (k_5 + i k_6 r) (h - h^*)' x$$

where  $k_2 < 0$ ,  $k_5 > 0$ ,  $k_6 < 0$ ,  $k_2$ ,  $k_4$ , and  $k_6$  are parameters (scalars),  $k_0$  and  $k_5$  are  $(1 \times n)$  vectors of parameters,  $a$  is an  $(1 \times m)$  vector of utility parameters that specifies the type of the consumer,  $k_1$  is an  $(m \times n)$  matrix of parameters,  $k_3$  is a  $(n \times n)$  matrix of parameters (symmetric and negative definite),  $h^* = h(A)$  or  $h(B)$  depending on whether the consumer is a member of Group A or B, and  $i$  is a  $(1 \times n)$  vector that is equal to  $(0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 1)$ .

### **The A-Contract Housing Market.**

The optimization problem of a consumer who buys an A-type housing contract is the following:

$$\max E_0 \sum_t b^t U(h_t, x_t, r_t, a)$$

with respect to  $h_t, x_t$  for all  $t = 0, 1, 2, 3, 4, \dots$

$$\text{subject to } y_t = P(h_t, r_t) + x_t$$

where  $y_t$  is income at  $t$ ,  $b$  is a discount rate, and  $E_0$  is the  $t=0$  period conditional expectation operator,  $P(h_t, r_t)$  is the hedonic price equation,  $P(h, r) = q_0 v' + q_1 r + (q_2 + i q_3 r) h'$ ,  $q_1$  and  $q_3$  are parameters (scalars),  $q_0$  and  $q_2$  are  $(1 \times n)$  vector of parameters, and  $v$  is a  $(1 \times n)$  vector that describes amenities of an urban area considered.

Substituting the utility function and the equilibrium price equation into the above optimization problem and solving it, I obtain that the demand for  $h$  is given by the following equation (note that the subscript  $t$  for time has been dropped from all variables to simplify the notation):

$$[h(y, a, r)]' = [b(r) + b(r)' - k_3]^{-1} [k_0' - k_4 q_2' + (b(r) - k_3) h(A)' + k_1' a' + c(r) + f(r) y]$$

where  $b(r) = (q_2' + i' q_3 r) (k_5 + i k_3 r)$

$f(r) = k_5 + i' k_6 r$  , and

$c(r) = -k_5' q_0 v' + (i' k_2 - k_5' q_1 - i' k_4 q_3 - i' k_6 q_0 v') r - i' k_3 q_1 r^2$

To illustrate the properties of the demand for  $h$ , I take the case that  $h = (h_1 \ h_2)$ ,  $q = (q_{21} \ q_{22})$ , where,  $h_1$  = size of the house,  $h_2$  = proximity to parks, rivers, forests, etc. It is assumed that  $q_{21} > 0$ ,  $q_1 > 0$  , and  $q_3 < 0$ . These imply the following:

$dh_1/dy > 0$  for every  $r$ , i.e., the high income consumers demand houses with more rooms independent of the value of the acid deposition variable  $r$ ,

$dh_2/dy > 0$  for relatively low acid deposition levels, and

$dh_2/dy < 0$  for relatively high acid deposition levels.

That is, high income consumers demand houses close to parks, rivers, etc, if there is not an acid deposition problem, and further from parks, rivers, etc, if there is an acid deposition problem that causes damages to parks, rivers, etc.

### **The B-Contract Housing Market.**

The optimization problem of a consumer who buys a B-type housing contract is the following:

$$\max E_0 \sum_t b^t U(h, x_t, r_t, a)$$

with respect to  $h, x_t$  for all  $t = 0, 1, 2, 3, 4, \dots$

$$\text{subject to } y_t = P(h, r_t) + x_t$$

Note that  $h$  does not carry a subscript  $t$ . The consumers of group B make their decisions at the beginning of  $t=0$  when the realization of  $r$  is assumed to be unknown to them. Moreover, it is assumed that 1)  $y_t = m(Y) + e_t$  for all  $t$ , 2)  $E_0 e_t = 0$  for all  $t$ , 3)  $E_0 e_0 = e_0$ , and 4)  $E_0 e_t r_t = 0$  for all  $t$  where,  $m(Y)$  is the mean income of consumers who buy a B-housing contract.

Substitute the utility function and the hedonic price equation into the above optimization problem to obtain that the demand for  $h$  is given by the following equation (under the assumption of rational expectations):

$$[h(y_0, m(Y), a, m(r), v(r))]' = [E_0 b(r_t) + E_0 b(r_t)' - k_3']^{-1} [k_0' - k_4 q_2' + (E_0 b(r_t) - k_3) h(B)' + k_1' a' + E_0 c(r_t) + (1-b) y_0 E_0 f(r_t) + b m(Y) E_0 f(r_t)]$$

where  $b(r)$ ,  $f(r)$ , and  $c(r)$  are defined in (2), (3), and (4),  $m(r) = E_0 r_t$ , and  $v(r) = E_0 (r_t)^2$ .

Using the example of the previous section to illustrate the properties of the demand for a B-type housing contract (recall that  $h_1$  is the demand for the size of a house and  $h_2$  is the proximity to parks, etc variable), I have that:

$dh_1/dm(Y) > 0$  and  $dh_1/dy_0 > 0$  for all  $m(r)$  values, i.e., high income consumers demand large size houses,

$dh_2/dy_0 > 0$  and  $dh_2/dm(Y) > 0$  for relatively low  $m(r)$  values, and

$dh_2/dy_0 < 0$  and  $dh_2/dm(Y) < 0$  for relatively high  $m(r)$  values.



That is, high expected acid deposition levels make the high income consumers demand houses that are located far from parks, rivers, etc (the opposite effect for low expected acid deposition levels).

### **The Equilibrium Price Equation.**

The optimum decisions of consumers and sellers depend on the equilibrium price equation. The price equation is determined so that buyers and sellers are perfectly matched. In equilibrium, no one of the economic agents can improve his position, all of their optimum decisions are feasible, and the price equation is determined by the distribution of consumer tastes, the supply for the differentiated product, and the income distribution.

PROPOSITION: The housing price<sup>5</sup> equation that equilibrates the markets described above can be approximated by the following equation:

$$P(h,r) = q_0 v' + q_1 r + (q_2 + i q_3 r) h'$$

where,

$$q_1 = (G_3 G_4 + G_3 G_5 G_1) + (G_3 G_5 G_2 + G_3 G_6) q_0'$$

$$q_2' = G_1 + G_2 q_0'$$

$$q_3 = (G_4 + G_5 G_1) + (G_5 G_2 + G_6) q_0'$$

$$q_0' = [G_7 G_2 + G_8 (G_5 G_2 + G_6) + G_9]^{-1} [G_{10} - G_7 G_1 -$$

$$G_8 (G_4 + G_5 G_1)]$$

$$G_1 = [k_5' h(A) + k_4]^{-1} (k_0' + k_1' m_A(a)' + k_5' m(y))$$

$$G_2 = - [k_5' h(A) + k_4]^{-1} k_5' v$$

$$G_3 = - i h(A)'$$

$$G_4 = (k_2 + k_6 m(y))/k_4$$

$$G_5 = - k_6 h(A)/k_4$$

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<sup>5</sup> The housing price is defined to be the gross rent and includes all maintenance costs.

$$G_6 = -k_6 v/k_4$$

$$G_7 = k_5' h(B) + i' k_6 m(r) h(B) + k_4$$

$$G_8 = k_5' i h(B)' m(r) + i' i h(B)' k_6 v(r) + i' k_4 m(r)$$

$$G_9 = k_5' v + i' k_6 m(r) v , \text{ and}$$

$$G_{10} = k_0' + i' k_2 m(r) + k_1' m_B(a) + (k_5' + i' k_6 m(r)) m(Y) ,$$

$m(y)$  is the mean income of the consumers of group A, and  $m_j(a)$  is the mean of  $a$  of the consumers of Group  $j$ , for  $j = A, B$ .

### Proof

Recall the equilibrium equations that are introduced in Section 2.2.:

$$d_t(h) = s_0(h) \text{ and } D_0(h) = S_0(h) \text{ for all } t, \text{ realizations of } r, \text{ and } h \text{ in } (a_0, b_0).$$

I take a first order approximation of i)  $s_0(h)$  around  $h(A)$ , ii)  $S_0(h)$  around  $h(B)$ , iii)  $d_t(h)$  around  $E_t h(y_t, a_t, r_t)$ , and iv)  $D_0(h)$  around  $E_0 h(y_0, m(Y), a, m(r), v(r))$ . To be more specific I take:

$$s_0(h) = s_1 + s_2 (h - h(A))'$$

$$S_0(h) = S_1 + S_2 (h - h(B))'$$

$$d_t(h) = d_1 + d_2 (h - E_t h(y_t, a_t, r_t))' , \text{ and}$$

$$D_0(h) = D_1 + D_2 (h - E_0 h(y_0, m(Y), a, m(r), v(r))).$$

Satisfaction of the equilibrium equations imply the identities:

$$d_1 = s_1, d_2 = s_2, D_1 = S_1, D_2 = S_2, \text{ and}$$

$$h(A) = E_t h(y_t, a_t, r_t)$$

$$h(B) = E_0 h(y_0, m(Y), a, m(r), v(r))$$

Satisfaction of (13) and (14) for all realizations of the acid deposition variable imply that the parameters of the price equation must satisfy (7) - (10).

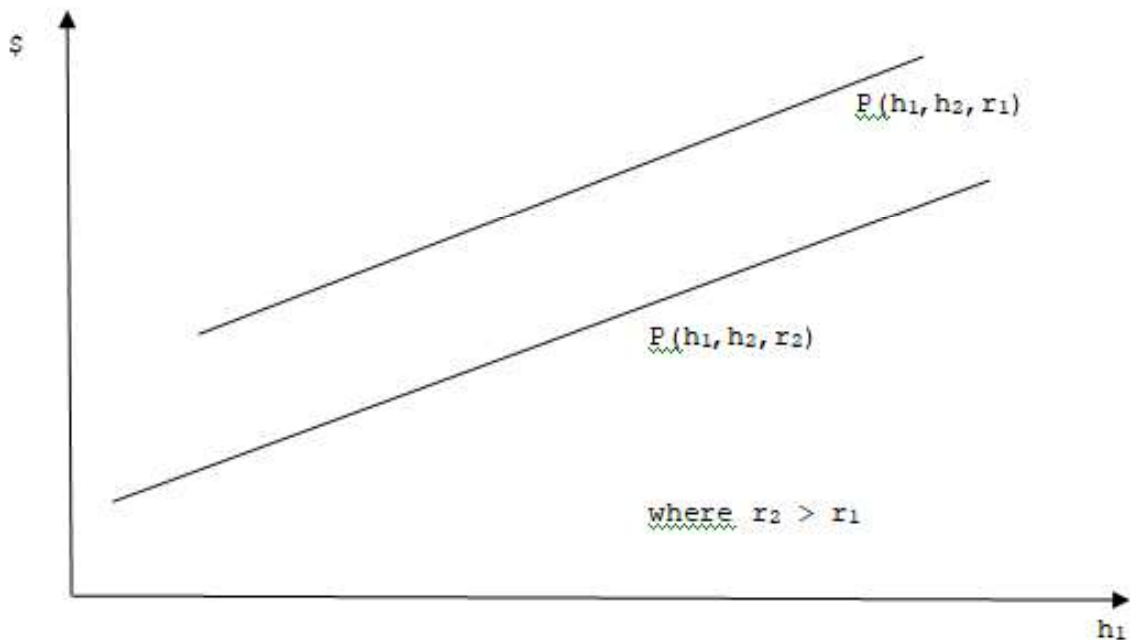
QED

Note that in a cross-section study  $r$ ,  $v$ , and the price equation parameters are going to be different in different urban areas.

It is expected:  $q_1 > 0$  and  $q_3 < 0$  since increases in acid deposition i) will tend to increase maintenance costs (damages to materials), and ii) will tend to increase the price of houses located further from parks, rivers, etc (places where consumers cannot see the effects of acid deposition). To illustrate the properties of the price equation, I take the example of Section 2.4 assuming  $q_{21} > 0$ . This implies:  $dP/dh_1 = q_{21} > 0$ ,  $dP/dr = q_1 + q_3 h_2$  and  $dP/dh_2 = q_{22} + q_3 r$ .

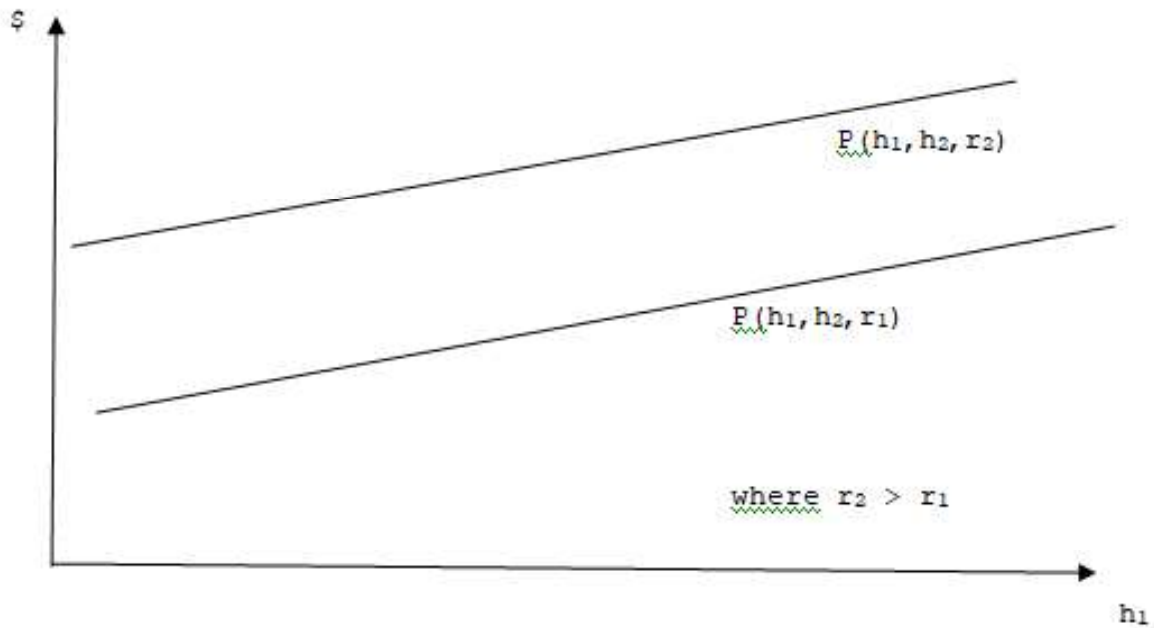
Now, I can draw Figures 1, 2, and 3 to illustrate the nature of the equilibrium price equation.

FIGURE 1



NOTE: Figure 1 refers to house with a high enough  $h_2$ -value, i.e., close enough to parks, rivers, forests, etc.

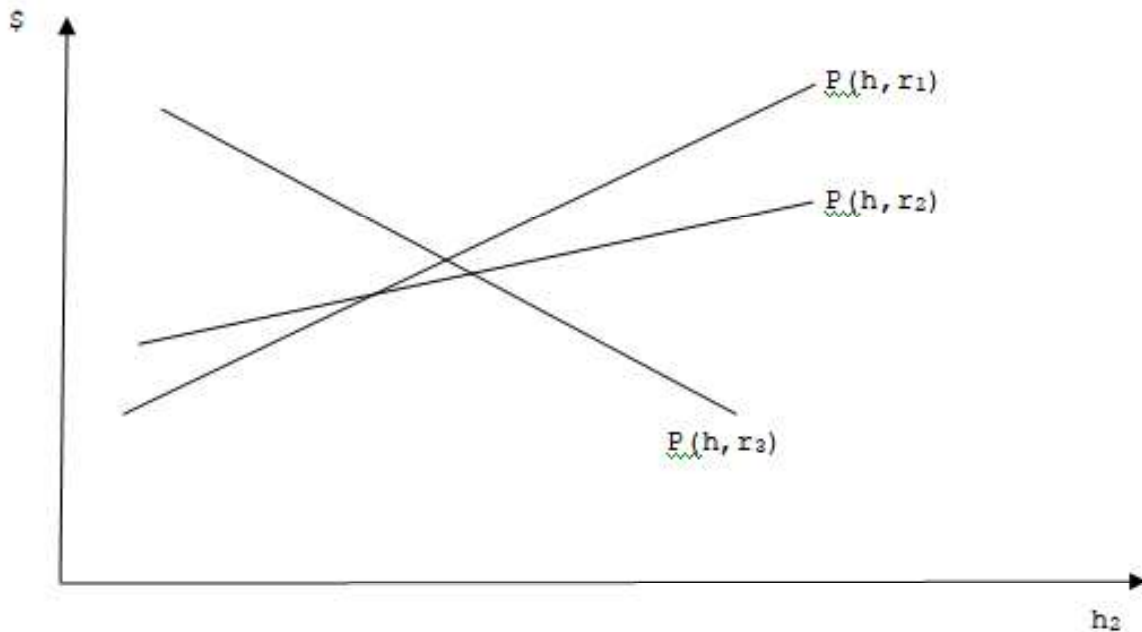
FIGURE 2



NOTE: Figure 2 refers to houses with a low enough  $h_2$ -value, i.e., far enough from parks, rivers, etc.

Figures 1 and 2 show that an increase in acid deposition decreases the rent of houses that are located relatively close to parks, rivers, etc, and increases the rental price of houses that are relatively far from parks rivers, etc.

FIGURE 3



where  $r_3 > r_2 > r_1$  and  $r_3$  is high enough.

Figure 3 shows that, as acid deposition increases, the rental price of houses relatively close to parks, rivers, etc decreases and the rental price of houses far from parks, rivers, etc increases. Moreover, for relatively high (low) acid deposition the rental price of a house that is closer to parks, rivers, etc is lower (higher) than the rental price of a house that is further from parks, rivers, etc.

**The complete model.**

The complete model consists of the following equations (1), (5), (6), (11), (12), and

$$V = b (q_0 v' + q_1 m(r)) / (1 - b) + (q_0 v' + q_1 r) + (q_2 + b i q_3 m(r)) h' / (1 - b) + i q_3 r h'$$

where,  $V = E_0 \sum_t b^t P(h, r_t)$  is the value at  $t=0$  of an  $h$ -house.

The last equation is added to complete the model because for households that buy a B-housing contract census tract housing statistics report the values of their houses. The parameters of the above set of equations that give the complete model must satisfy equations satisfy (7) - (10).

The model is linear in consumer income (current and expected) and the vector of utility parameters,  $a$ , that specifies the type of the consumer. This solves the aggregation problem and indicates that it is legitimate to use census tract data to estimate and test the model.

The model can be estimated using census tract data and the cross equation restrictions can provide a test of the theory. Given the estimation results willingness to pay can be subsequently computed by proper integration of the demand equation(s).

### **Conclusions.**

This paper presents a theory that is appropriate for testing whether there are effects of acid deposition on household of residential areas. The theoretical model can be estimated and the internal consistency of the theory can be tested. To estimate the model there is need for cross-section or time series data.

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