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RESEARCH NOTE

A NOTE ON STOPPING STRATEGY OF AUCTION: MAXIMIZATION SELLER GAIN PERSPECTIVE

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ABSTRACT

In the current paper, the binomial tree is used to derive the optimal stopping time for seller of a commodity in a specified auction. It is assumed that the bids signals have valuable information to make decision about the best time for selling the commodity while using the derived optimal strategy the seller gain is maximized. It is seen that it is the time point at which a portfolio containing a bond and a call option gets its maximum. Some examples are given to show the value of theoretical results.

Keywords: Auction, Binomial tree, Bond, Call option, Stopping strategy

JEL Classification: D44, G11, G12

Introduction

In competitive global markets, such as online auctions, to achieve profitable growth and stand out among competitors, it is necessary to compete strategically. Most people attend auctions without a bidding strategy. The good news is you may win an auction without having reached your predetermined maximum price. The bad news is you may be the under-bidder at a few auctions before you win one. But there are bidding strategies that you can adopt to give yourself every chance of winning an auction. While you may have watched many auctions, there is a different element at play when you are bidding. Auctions need competitive bidding to perform. As a vendor, it is so simple to ask a friend to register and bid up to the reserve price, creating the illusion that the property is in demand. The simple advice for buyers is to avoid bidding unless the property has met or exceeded reserve. In the past, less skilled auctioneers would call the property on the market as soon as it hit reserve. Skilled auctioneers won't immediately advise the crowd when the auction has been met. A good aggressive bidder will have no qualms in asking the auctioneer if the property is on the market yet. You may or may not get an answer to that question, but it is worth asking and continuing to ask during the auction.

There is a vast literature in bidding strategies in an auction. For example, Tesauro and Das (2001) considered the high-performance strategies of bidding for the continuous double auction. Goldman et al. (2003) studied experimental equilibrium strategies for selecting sellers and satisfying buyers. He et al. (2003) studied a fuzzy logic based bidding strategy for autonomous agents in continuous double auctions. Ivashina and Kovner (2011) considered the role of bidding in auctions in private equity of leveraged buyout firms. Hattori et al. (2013) proposed a dynamic programming model for determining bidding strategies in sequential auctions. They used a quasi-linear utility and considered budget constraints. Hege et al. (2013) considered the asset sales via auctions and the role of buyers. The role of strategic buyers versus private equity studied. Dutting et al. (2017) studied optimal auctions through deep learning. Abeille and Calauz (2018) proposed explicit shading strategies for repeated truthful auctions. This paper tells some points about stopping strategies in an auction game from stochastic stopping time perspectives which are fundamental building blocks of martingale theories and are widely used in pricing American options.

Practical algorithm

Here, a practical algorithm is proposed for accepting or rejecting bids by seller. To this end, notice that as soon as bidders are arrived to seller in auction, it is a critical decision to believe a specific bid or to continue. Almost all commodities and financial assets are sold in auctions. Guo (2002), using a binomial tree structure, proposed an optimal strategy for sellers in online auctions. In this paper, we also study the problem of seller stopping strategy at which the seller gain is maximized.

Following Guo (2002) notations, let $\{z_t, t \geq 1\}$ be a sequence of iid random variables such that

$$z_t = \begin{cases} u & \text{with probability } p \\ d & \text{with probability } 1-p, \end{cases}$$

where

$$0 < d < 1 < u, 0 < p < 1$$

and received bids at time $t \geq 1$ are $x_t = x_0 \prod_{i=1}^{t} z_i$. Here, x_0 is initial bid at time zero and set $M_t = \log(x_t)$. The transaction cost is r and L is the lowest price of seller of auction.

Here, using the binomial tree model, the seller gain is formulized and a recursive formula is obtained. Indeed, seller gain at time t is $G_t = e^{-rt}\max(x_t, L) = e^{-rt}y_t$. It is interested to find a stopping time like τ to maximize

$$E(e^{-r\tau}\max(x_\tau, L)).$$

Notice that $y_t = L + \max(x_t - L, 0)$. That is y_t is the payoff of a bond with face value L and a call option with strike price L and maturity t. Under the no arbitrage assumption, the risk neutral probability measure is $p = \frac{e^r - d}{u - d}$ and expectations are taken under this probability measure. Also, notice that

$$G_t = e^{-rt}L + e^{-rt}E_p(\max(x_t - L, 0)).$$

The second term is obtained using the Black-Scholes formula. Finding τ is equivalent to finding the first time point (stopping time) at which a portfolio receives to its maximum during its living period. The necessary condition to this situation is $\frac{\partial G_t}{\partial t} = 0$. Therefore,

$$\frac{\partial G_t}{\partial t} = -re^{-rt} + \theta_t = 0,$$

where θ_t is the Greek letter which is the derivative of Black-Scholes formula with respect to time. Then, ϑ is the root of above equation, then using the Newton-Raphson method, there is a sequence of ϑ_n such that

$$\vartheta_{n+1} = \vartheta_n - \frac{\frac{\partial G_t}{\partial t}}{\frac{\partial^2 G_t}{\partial t^2}}\Big|_{t=\vartheta_n}.$$

The following proposition summarizes the above discussion.

Proposition

The Newton-Raphson solution of G_t is given by

$$\vartheta_{n+1} = \vartheta_n - \frac{\frac{\partial G_t}{\partial t}}{\frac{\partial^2 G_t}{\partial t^2}}\Big|_{t=\vartheta_n}.$$

Remark

The sensitivity of G_t with respect to change of parameters L, t, x, r can be found by Greek letters of call option which is achievable by running Monte Carlo simulations.

As follows the abovementioned practical algorithm is proposed:
(a) First, parameters of binomial tree such as u, d, p are obtained.
(b) Using the Black-Scholes formula and, the present value of a portfolio containing a call option and a bond is calculated.
(c) The threshold K is computed using a Monte Carlo simulation and as soon as G_t is below the threshold, the seller doesn't accept to sell, however, as soon as G_t passes the K and takes its maximum, then it is the time to sell the commodity or financial asset.

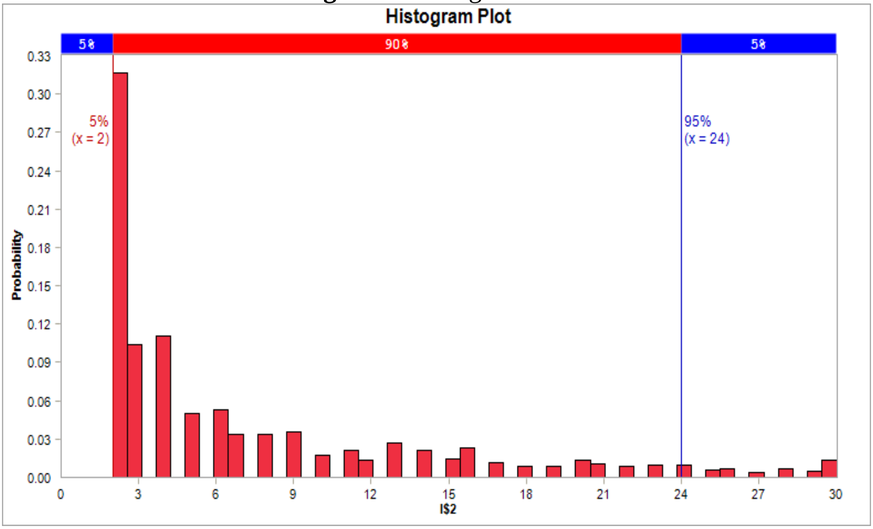
Simulations

Here, throughout some simulated examples, the empirical distribution of $\vartheta = \text{argmax}(e^{-rt} y_t)$ is simulated using the *Model-Risk* add-in of Excel.. To this end, for the first example,

Example 1

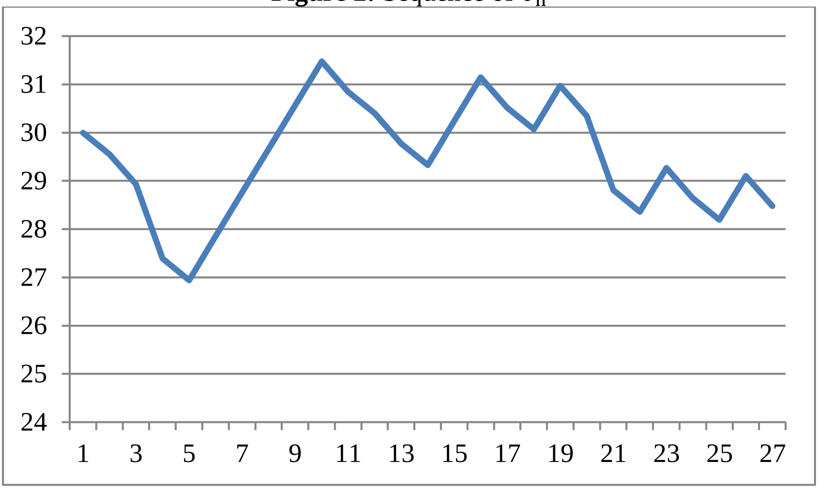
Let $p = 0.3, u = 3, d = 0.5, L = 1.5, x_0 = 8$ and $r = 0.05$. Then, the following figure provides the empirical distribution of ϑ.

Figure 1: Histogram of ϑ



The following figure shows the convergence of ϑ_n to actual value of ϑ. Visually, it is seen that the rate of convergence is good.

Figure 2: Sequence of ϑ_n



Here, threshold K is given such that $P(max_t G_t > K) = \alpha$. The following table gives these values

Table 1: Values of K

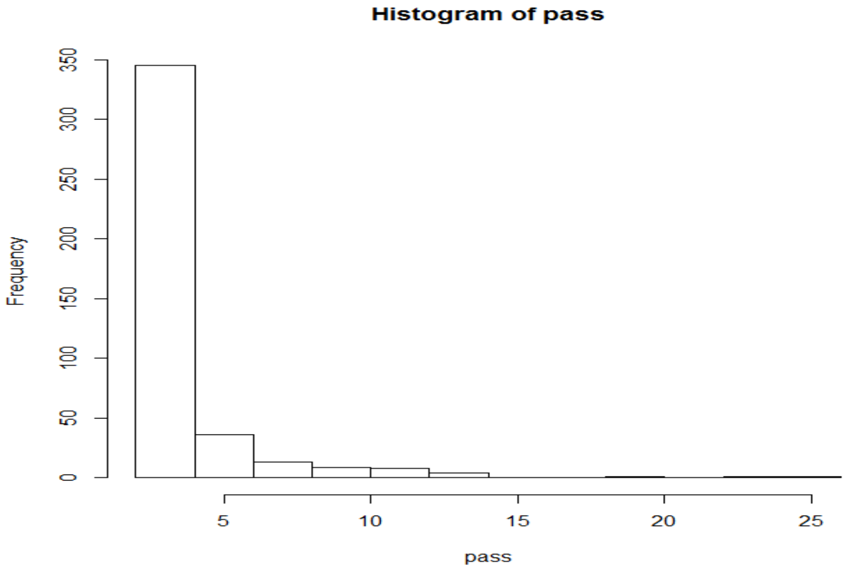
α	0.1	0.05	0.025	0.01
K	0.25	0.85	2.33	7.23

That is the empirical distribution of maximize of $G_t$is a right-skewed distribution which implies that sellers will sell commodity or financial asset as soon as he receives an acceptable bid. Indeed, they will not wait until the $G_t$attains its maximization. This fact shows that sellers will miss more benefit.

Example 2

The main theoretical hypothesis of the current paper is that "The current method has high performances in detecting suitable time for bidding in auction". Indeed, this type of strategic thinking about bidding decision in auctions provides upper winning probabilities. To this end, we provide empirical support for our hypothesis by demonstrating a practical example. Suppose that the lower bound of seller for a given commodity is $L = 30$. Assume that $u = 2.5, d = 0.4, x_0 = 27$. Here, the minimum time at which seller should be considered for selling the commodity is $\tau = \inf\{t, x_t > L_u\}$, where L_u is the upper bound at which seller will not wait for another bid and will buy the commodity. Here, it is supposed that $L_u = 30$. Notice that the natural logarithm of bid signals behave like a random walk satisfying $\frac{\ln(x_t)}{0.9163} = \frac{\ln(x_{t-1})}{0.9163} + \delta_t$, where $\delta_t = 1$ with probability of $p = 0.3$ and -1 with probability of 0.7. Here, 0.9163 is the natural logarithm of u. Then, $\tau = \inf\{t, \frac{\ln(x_t)}{0.9163} > 3.712\}$. Simulated results show that with probability of 0.6 sellers will not receive to L_u and if he/she passes the desired threshold then the mean and standard deviation of τ are given by 3.5, 3, respectively. The following figure shows the histogram time point at which bid signals pass the threshold.

Figure 3: Histogram of pass time point



Conclusions

This section is proposed to emphasis in economic meaning of the results. To this end, notice that in some economics, a wide variety of institutions have emerged for determining prices and conducting trade. In retail stores, the price of each good is usually posted by the seller and individual buyers can do little to influence that price. Each type of various arrangements for conducting trade has some merits. For example, detailed negotiations provide a flexible way to determine prices, product features and financing terms.

Auctions have properties that place sellers and buyers somewhere between negotiations and posted prices. Billions of dollars of US treasury bills are sold each week using a sealed-tender auction. These types of facilities make prices to reflect current demand conditions and to respond to new information about variables like the money supply. Because of various kinds of auctions, such as Dutch, English, discriminatory, and Vickrey auctions, taking profitable strategies in bidding is too important. In this paper, the bidding strategy is introduced based on binomial tree structure and using a stopping time tools to make sure certain profitable time points for bidding. This point shows the importance of the results of the current paper.

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